Hw 4

Read chapters 6 and 7 of the textbook.

Main skills.

- you need to understand the notion of set and subset
- given a universe U and sets A and B you need to know how to build $A \cup B$, $A \cap B$, A- B, P(A), A^c and $A \times B$.
- you need to know how to prove that two sets are equal.
- you need to know how to prove that a set is a subset of given set.
- work with quantifiers, in particular, you need to know how to negate a sentence containing quantifiers and you need to know how to prove a statement of the form $\forall x \exists y Q(x, y)$ or $\exists x \forall y Q(x, y)$.

Do the following problems from your textbook:

- p. 72: 6.5
- p 72: 6.6
- p 87 : 7.7
- p. 116:7
- p. 117: 11

. Do the following additional problems.

1. Give a necessary and sufficient condition on sets A and B for A-B=B-A to hold. Prove your answer.
2. For each of the following statements do the following:
   - translate it into a symbolic statement.
   - write the negation in symbolic form without using the connective "not" ($\neg$)
   - write the negation in plain English.
   - prove or disprove the statement.

   You can use " odd x " to stand for x is odd and " even x " to stand for x is even x div y " to stand for " x divides y" and " x notdiv y " to stand for " x does not divide y"

   (a) For all integers x there is an integer y that is smaller than x.
   (b) Every integer greater than 4 is odd.
   (c) There is an integer less than 10 that belongs to all elements of P(Z).

(d) There is an integer divisible by 2 and 3 and not by 6.

3. Circle all the statements below that are equivalent to the negation of the statement "for all integers x, if x has property P then either x is even or there exists an integer y such that y is greater than x and y does not have property Q. No justification needed.

   (a) For all integers x, x does not have property P implies either x is even or there exists an integer y such that y is greater than x and y does not have property Q.

   (b) There is an integer x such that if x has property P then x is odd and for all integers y either y is less than or equal to x or y has property Q.

   (c) There is an integer x such that x has property P and x is odd and for all integers y y is less than or equal to x and y has property Q

   (d) there is an integer x such that x is not even and for all y y is less than x and y has property Q implies x does not have property P.

   (e) There is an integer x such that x has property P or x is odd and for all integers y y is less than or equal to x and y has property Q.

   (f) There is an integer x such that x has property P and x is odd and for all integers y either y is less than or equal to x or y has property Q.

4. Decide if the following statements are true for every set A and B (no proof necessary)

   (a) A $\cap$ B $\subseteq$ A $\cup$ B

   (b) A - B $\subseteq$ A $\cap$ B

   (c) $\emptyset \in$ A

   (d) $\emptyset \subseteq A$

   (e) $\{\emptyset\} \in$ P(A)

   (f) $\{A\} \subseteq A$

   (g) $A \subseteq \{A\}$

   (h) $\{A\} \subseteq \{A\}$

5. Let A and B be sets. Prove that $A \subseteq B \Leftrightarrow P(P(A)) \subseteq P(P(B))$

6. Let A_i, $i \in Z^+$ be sets is some universe U, prove that $(\bigcup_{i=0}^{\infty} A_i)^c = \bigcap_{i=0}^{\infty} A_i^c$