Hw 3

Read chapter 5 (again)

Main skills. You need to know how to :

- do a proof by (regular, strong or modified) induction.
- you need to understand the notion of definition by recursion.

Do the following problems from your textbook:
p.54:12
p.56:20
Do the following additional problems.

1. Consider the following table

$$\begin{array}{ccccccccc} & & & 3^0 & & & & = & 1(\text{ note } 3^1 = 3) \\ & & 3^0 & + & 3^1 & & & = & 4(\text{ note } 3^2 = 9) \\ & 3^0 & + & 3^1 & + & 3^2 & & = & 13 \text{ (note } 3^3 = 27) \\ 3^0 & + & 3^1 & + & 3^2 & + & 3^3 & = & 40 \text{ (note : } 3^4 = 81) \end{array}$$

   Guess a formula for the sum $3^0 + 3^1 + 3^2 \cdots + 3^n$ (that is guess something like $\sum_{i=0}^{n} 3^i$= expression in n) and prove your guess by induction.

2. Prove that if r is a real number with $|r| < 1$ then $\displaystyle\sum_{i=0}^{n-1} r^i = \frac{1-r^n}{1-r}$

3. Prove that every positive integer n can be expressed as the product of an odd number and a power of 2, that is, for every $n \geq 1$ there are h in Z^+, h odd and k in Z, $k \geq 0$ such that $n = h \cdot 2^k$. HINT: use strong induction.

4. Given the recursive definition:

   $a_1 = 1$

   $a_2 = 9$

   $a_{n+1} = 6a_n - 9a_{n-1}$ for $n + 1 \geq 3$

   Prove that $\forall n \geq 1 \quad a_n = (2(n-1)+1)3^{n-1}$.

5. Prove that for all $n \geq 1$ $\displaystyle\sum_{i=0}^{n-1} u_{2i+1} = u_{2n}$ where u_n is the n-th Fibonacci number.

6. Prove that there are u_{n+1} (the n+1 Fibonacci number) different ways to tile a 1xn board using squares (i.e. 1x1 tiles) and dominoes (i.e. 1x2 tiles).

7. The statement $\forall n \geq 0\,, 11n = 0$ is false, therefore any proof of it must be wrong. What is wrong with the following proof ?

   Proof by strong induction:

   Base case : if $n = 0$ then 11 n =0

   Induction step : assume $11\,t = 0$ for $t = 0, 1, \ldots k$, we need to prove $11(k+1) = 0$. Choose $i, 0 \leq i \leq k$ and let $j = (k+1) - i$ then $k+1 = i+j$ and $11(k+1) = 11\,i + 11\,j = 0$ by induction assumption.

8. Consider the following sequence $a_{n\,m}$ where

$$a_{n\,1} = 1 \text{ for all } n \in N$$

$$a_{1\,m} = 0 \text{ for all } m \geq 2$$

$$a_{n+1\,m+1} = a_{n\,m} + a_{n\,m+1} \text{ for all } n,\, m \in N$$

   Use induction to prove that $\forall n \in N (x+y)^n = \sum_{i=0}^{n} a_{n+1\,i+1}\, x^{n-i} y^i$