

Read chapters 8 and 9 of the textbook

Main skills. You need to understand:

- The definition of function
- how to decide if a function is injective, surjective or bijective
- the definition of composition.
- The definition of inverse  $f^{-1}$  of a function  $f$ .

Do the following problems from your textbook:

- p. 99: 8.2
- p 113: 9.1
- p 118: 19

. Do the following additional problems.

1. Define a function  $f : Z \rightarrow Z$  by:

$$f(x) = \begin{cases} n + 1 & \text{if } n \text{ is even} \\ n + 2 & \text{if } n \text{ is odd} \end{cases}$$

- (a) Is  $f$  injective ? Prove your answer.
- (b) Is  $f$  surjective ? Prove your answer.

2. Define a function  $f : Z \rightarrow Z$  by  $f(x) = x + 5$ .

- (a) Compute  $f^2(x) = f(f(x))$
- (b) Compute  $f^3(x)$
- (c) Find and prove a formula for  $f^n(x)$ .

3. Let  $d : Z^+ \rightarrow Z^+$  be defined by  $d(x)$ = number of positive divisors of  $x$ . For example  $d(6) = 4$  because 6 is divisible by 1, 2,3,and 6.

- (a) Describe the set  $S = \{x \in Z^+ | d(x) = 2\}$ .
- (b) Is  $d$  injective ?
- (c) Make a conjecture for a formula for  $d(2^n)$  (i.e write  $d(2^n) =$  formula in  $n$ ) for  $n \geq 0$  and prove your formula .
- (d) is  $d$  surjective ?

4. Give examples of functions  $f A \rightarrow B$  and  $g B \rightarrow C$  that satisfy the following conditions, or explain why no example exists:

- (a)  $g$  surjective  $g \circ f$  not surjective
- (b)  $g$  injective  $g \circ f$  not injective.
- (c)  $f$  not injective  $g \circ f$  injective.