1. Circle all the statements below that are equivalent to the negation of the statement "for all integers x, if x has property P then x is divisible by 2 and x is divisible by 3". No justification needed.

(a) For all integers x, x has property P and x is not divisible by 2 and 3.
(b) There is an integer x which is not divisible by 2 and it is not divisible by 3 and has property P.
(c) For all integers x, if x has property P then x is not divisible by 2 or x is not divisible by 3.
(d) There is an integer x such that if x has property P then x is not divisible by 2 and by 3.
(e) There is an integer x which has property P and either it is not divisible by 2 or it is not divisible by 3.
(f) There is an integer x such that if x has property P then x is not divisible by 2 or x is not divisible by 3.

2. Define a function \( f : \mathbb{Z} \rightarrow \mathbb{Z} \) by:

\[
f(x) = \begin{cases} 
2x & \text{if } x \geq 0 \\
3x & \text{if } x < 0
\end{cases}
\]

Is \( f \) injective? Is it surjective?
What about \( g : \mathbb{Z} \rightarrow \mathbb{N} \) defined by:

\[
g(x) = \begin{cases} 
x & \text{if } x \geq 0 \\
-3x & \text{if } x < 0
\end{cases}
\]

Is \( g \) injective? Is it surjective?

Sets:

3. Let A and B be non empty sets. Show that \( A \times B = B \times A \) if and only if \( A = B \).

4. Show that \( (A \cup B) \subseteq (B \cap C) \Rightarrow A \subseteq C \).

5. Show that \( P(A \cap B) \subseteq P(A) \cap P(B) \)

Induction:

6. Consider the following game: two players each start with a stack of \( n \) quarters. At each turn player 1 chooses one stack and removes some coins from the stack. Player two then removes some coins (possibly a different number of coins from player 1) from the other stack. The player that removes the last coin(s) wins. Show that for any \( n > 0 \) player 2 can win the game that starts with two stacks of \( n \) coins each.

7. Prove that \( \sum_{i=1}^{n} i^3 = \left( \frac{n(n+1)}{2} \right)^2 \)
8. Show that the number of strings of 0’s and 1’s of length \( n \) that do not contain two consecutive 1’s is \( u_{n+2} \)

9. Given the sequence, defined recursively by:
   
   \[
   a_1 = 1 \\
   a_2 = 1 \\
   a_3 = 2 \\
   a_{n+1} = a_n a_{n-1} a_{n-2} \text{ for } n + 1 \geq 4
   \]
   
   prove that \( \forall n \geq 9 \quad a_n \geq 2^{2n} \)