Math 125 Section D (Pezzoli)
Fall 2017
Midterm #2

Name ______

TA: ______

Section: ______

.

- Your exam contains 4 problems. The entire exam is worth 55 points.
- You have 70 minutes to complete this exam.
- This exam is closed book. You may use one $8\frac{1}{2}$" $\times$ 11" sheet of notes (both sides). Do not share notes.
- The only calculator allowed is the TI 30x IIS.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. Show your work in evaluating integrals, even if they are on your note sheet.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- Give your answer in exact form means write $\ln(2) - \pi$ instead of -2.45. If you are asked to give a decimal approximation, use two decimal digits in your final answer.
- This exam has 5 pages, including this cover sheet. Please make sure that your exam is complete.

Problem #1(20 pts) ______

Problem #2(10 pts) ______

Problem #3(13 pts) ______

Problem #4(12 pts) ______

TOTAL (55 pts) ______

1. Calculate the following integrals.

(a) $\int_4^5 \frac{x}{x^2-x-2}\,dx$. Give your final answer as a decimal.

$x^2 - x - 2 = (x-2)(x+1)$; $\frac{x}{x^2-x-2} = \frac{A}{x-2} + \frac{B}{x+1}$ if

$A(x+1) + B(x-2) = x$; if $x = -1$ we get $-3B = -1$ $B = 1/3$

if $x = 2$ we get $3A = 2$ $A = \frac{2}{3}$

$$\int_4^5 \frac{\frac{2}{3}}{x-2} + \frac{\frac{1}{3}}{x+1}\,dx = \frac{2}{3}\ln|x-2| + \frac{1}{3}\ln|x+1| \Big|_4^5 = \frac{2}{3}\ln\left(\frac{3}{2}\right) + \frac{1}{3}\ln\left(\frac{6}{5}\right)$$

$$\frac{1}{3}\ln\left(\frac{54}{20}\right) \approx \boxed{0.33}$$

(b) $\int \sqrt{3-2x-x^2}\,dx$

$x^2 + 2x - 3 = (x+1)^2 - 4$; $\int \sqrt{4-(x+1)^2}\,dx$

$u = x+1$

$du = dx$

$\int \sqrt{4-u^2}\,du$

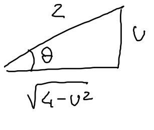
$u = 2\sin\theta$

$du = 2\cos\theta\, d\theta$

$$\int 2\cdot 2\cos^2\theta\, d\theta = 4\int \frac{1}{2}(1+\cos 2\theta)\,d\theta = 2\theta + \sin(2\theta) + C$$

$$= 2\theta + 2\cos\theta\sin\theta + C$$

going back to u



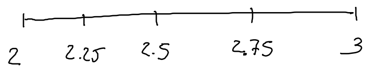
$\cos\theta = \frac{\sqrt{4-u^2}}{2}$

$$2\,\text{arc}\sin\left(\frac{u}{2}\right) + \frac{2\sqrt{4-u^2}}{2}\cdot\frac{u}{2} + C$$

going back to x

$$2\,\text{ersin}\left(\frac{x+1}{2}\right) + \sqrt{4-(x+1)^2}\cdot\frac{(x+1)}{2} + C$$

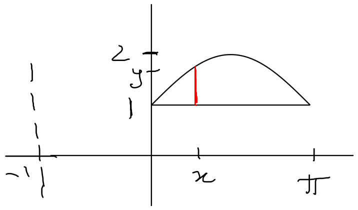
2. Consider the integral $I = \int_2^3 \overset{= f(x)}{\frac{\sin x}{ln(x)}}\, dx$. Use $n = 4$ subdivisions and Simpson rule to approximate I. Show your work and give your final answer as a decimal.



$\Delta x = \frac{1}{4}$

$\left[f(2) + 4f(2.25) + 2f(2.5) + 4f(2.75) + f(3)\right] \cdot \frac{1}{12} \approx 0.67$

3. Below is the graph of the region R bounded by the curves $y = \sin x + 1$, $y = 1$ between $x = 0$ and $x = \pi$.



Find the volume of the solid of revolution obtained by rotating the region R around the line $x = -1$.

$\Delta V = 2\pi r\, h\, \Delta x \qquad r = x+1, \qquad h = y-1 = \sin x$

$$V = 2\pi \int_0^{\pi} \underset{u}{(x+1)}\, \underset{dv}{\sin x}\, dx = -2\pi(x+1)\cos x \Big|_0^{\pi} + 2\pi\int_0^{\pi} \cos x\, dx =$$

$$-2\pi(x+1)\cos x \Big|_0^{\pi} + 2\pi \sin x \Big|_0^{\pi} = 2\pi^2 + 4\pi$$

Using washers $\Delta V = \pi(r_2^2 - r_1^2)$ $\qquad r_2 = \pi + \arcsin(y-1)$, $r_1 = \arcsin(y-1)$

4. A bag of sand weights 100 lb. The bag of sand is being lifted up from the ground. The bag has a hole and loses sand at a constant rate r lb/ft, that is after being lifted one foot, the bag has lost r lb of sands, after two feet the bag has lost 2r lb of sand and so on . The work done to lift the bag 10 feet is 950 lb-ft. Find r.

weight x feet above ground $100 - rx$

work to lift bag from x_i to $x_{i+1} = x_i + \Delta x$ feet above the ground is $\Delta W = 100 - rx_i \, \Delta x$

$W = \int_0^{10} 100 - rx \, dx = 950$ solve for r

$100x - r\frac{x^2}{2}\Big|_0^{10} = 1000 - r \cdot 50 = 950$ $\boxed{r = 1 \text{ lb/ft}}$

Math 125 Section D (Pezzoli)
Fall 2017
Midterm #2
.

Name ____________

TA: ____________

Section: ____________

- Your exam contains 4 problems. The entire exam is worth 55 points.
- You have 70 minutes to complete this exam.
- This exam is closed book. You may use one $8\frac{1}{2}" \times 11"$ sheet of notes (both sides). Do not share notes.
- The only calculator allowed is the TI 30x IIS.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. Show your work in evaluating integrals, even if they are on your note sheet.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- Give your answer in exact form means write $\ln(2) - \pi$ instead of -2.45. If you are asked to give a decimal approximation, use two decimal digits in your final answer.
- This exam has 5 pages, including this cover sheet. Please make sure that your exam is complete.

Problem #1(20 pts) ____________

Problem #2(10 pts) ____________

Problem #3(13 pts) ____________

Problem #4(12 pts) ____________

TOTAL (55 pts) ____________

1. Calculate the following integrals.

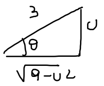
(a) $\int \sqrt{8-2x-x^2}\,dx$

$x^2+2x-8=(x+1)^2-9$ $\quad \int \sqrt{9-(x+1)^2}\,dx$ $\quad u = x+1$, $du = dx$ $\quad \int \sqrt{9-x^2}\,du$ $\quad u = 3\sin\theta$, $du = 3\cos\theta\,d\theta$

$$\int \sqrt{9(1-\sin^2\theta)}\;3\cos\theta\,d\theta = 9\int \cos^2\theta\,d\theta = \frac{9}{2}\int 1+\cos 2\theta\,d\theta = \frac{9}{2}\theta + \frac{9}{4}\sin 2\theta + C$$

$$\frac{9}{2}\theta + \frac{9}{2}\sin\theta\cos\theta + c$$

going back to u



$$\frac{9}{2}\arcsin\left(\frac{u}{3}\right) + \frac{9}{2}\,\frac{u}{3}\cdot\frac{\sqrt{9-u^2}}{3}$$

going back to x $\quad \frac{9}{2}\arcsin\left(\frac{x+1}{3}\right) + \frac{x+1}{2}\sqrt{9-(x+1)^2} + C$

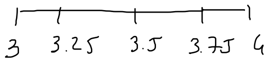
(b) $\int_3^4 \frac{x}{x^2+x-2}\,dx$. Give your answer as a decimal.

$x^2+x-2=(x-1)(x+2)$, $\quad \frac{x}{x^2+x-2} = \frac{A}{x-1} + \frac{B}{x+2}$ $\quad$ if $\quad A(x+2)+B(x-1)=x$

if $x=-2$ $\quad -3B=-2$ $\quad B=\frac{2}{3}$, $\quad$ if $x=1$ $\quad 3A=1$ $\quad A=\frac{1}{3}$

$$\int_3^4 \frac{\frac{1}{3}}{x-1} + \frac{\frac{2}{3}}{x+2}\,dx = \frac{1}{3}\ln|x-1| + \frac{2}{3}\ln|x+2|\Big|_3^4 = \frac{1}{3}\ln\left(\frac{3}{2}\right) + \frac{2}{3}\ln\left(\frac{6}{5}\right) \approx 0.26$$

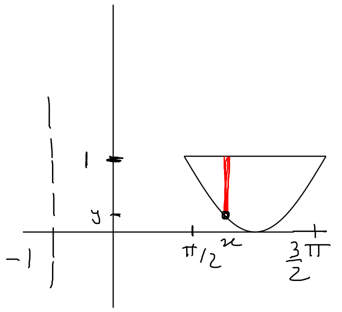
2. Consider the integral $I = \int_3^4 \overset{= f(x)}{\frac{\ln x}{e^x}}\, dx$. Use $n = 4$ subdivisions and Simpson rule to approximate I. Show your work and give your final answer as a decimal.



$\Delta x = \frac{1}{4}$

$$S_4 = \frac{1}{12}\left(f(3) + 4f(3.25) + 2f(3.5) + 4f(3.75) + f(4) \right) \approx 0.04$$

3. Below is the graph of the region R bounded by the curves $y = \cos x + 1$, $y = 1$ between $x = \frac{\pi}{2}$ and $x = \frac{3}{2}\pi$.



Find the volume of the solid of revolution obtained by rotating the region R around the line $x = -1$.

$\Delta V = 2\pi r h \, \Delta x \qquad r = x + 1 \, , \quad h = 1 - y = 1 - (\cos x + 1) = -\cos x$

$V = -2\pi \int_{\pi/2}^{\frac{3}{2}\pi} (x+1) \cos x \, dx = -2\pi \int_{\pi/2}^{\frac{3}{2}\pi} \underset{u}{x} \, \underset{dv}{\cos x \, dx} - 2\pi \int_{\pi/2}^{\frac{3}{2}\pi} \cos x \, dx$

$-2\pi \; x \sin x \Big|_{\pi/2}^{3/2\pi} + 2\pi \int_{\pi/2}^{\frac{3\pi}{2}} \sin x - 2\pi \sin x \Big|_{\pi/2}^{\frac{3\pi}{2}} = 2\pi \frac{3}{2}\pi + 2\pi \frac{\pi}{2} + 2\pi \cos x \Big|_{\pi/2}^{3/2\pi} + 4\pi$

$= 4\pi^2 + 4\pi$

Using washers $\Delta V = \pi(r_2^2 - r_1^2) \, \Delta y \qquad r_2 = \pi + \arccos(y-1) \qquad r_1 = \arccos(y-1)$

4. A bag of sand weights 100 lb. The bag of sand is being lifted up from the ground. Since it is raining the bag gets heavier and heavier. Its weight increases at a constant rate r lb/ft, that is after being lifted one foot, the bag has gained one extra r lb of weight, after two feet the bag has gained 2r extra lb of weight and so on . The work done to lift the bag 5 feet is 525 lb-ft. Find r.

Weight of the bag x feet above the ground is $100 + rx$

Work done to lift bag from x_i to $x_i + \Delta x$ feet is $\Delta W = (100 + r x_i)\Delta x$

$W = \int_0^5 100 + rx \, dx = 525$ solve for r

$100x + r\frac{x^2}{2}\Big|_0^5 = 500 + r\,12.5 = 525$ $r = 2$ lb/ft