# Lesson3

Read 5.2

General Riemann sum, midpoint rule

The definite integral

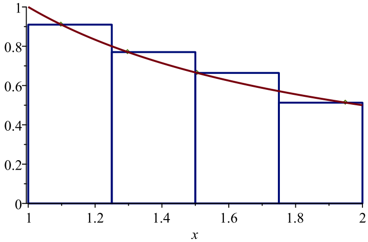
Consider a function $f$ over an interval $[a, b]$ .

- $n$ number of subdivisions.
- $\Delta x = \frac{b-a}{n}$.
- $x_0 = a$, $x_i = a + i\Delta x$ for $i = 1$ to $n$. (note $x_n = b$).
- $x_i^*$ any point in $[x_{i-1}, x_i]$

General Riemann sum:

$$\sum_{i=1}^{n} f(x_i^*)\Delta x =$$

$$= f(x_1^*)\Delta x + f(x_2^*)\Delta x + \cdots + f(x_n^*)\Delta x$$




An approximation of $\int_1^2 f(x)\,dx$ with randomly selected points, where $f(x) = \frac{1}{x}$ and the partition is uniform. The approximate value of the integral is 0.7143039474. Number of subintervals used: 4.

What happens when $n$ gets bigger and bigger ?

### Def

The definite integral of $f$ from $a$ to $b$ is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} = \sum_{i=1}^{n} f(x_i^*)\Delta x$$

provided the limit exists and gives the same value for all possible choices of the $x_i^*$. In this case we say $f$ is integrable on $[a, b]$

### Theorem

If $f$ is continuous on $[a, b]$ or $f$ is bounded and $f$ has only a finite number of discontinuities then it is integrable

In this class we will learn

- To calculate definite integrals.
- To approximate definite integrals.
- To use definite integrals in applications.

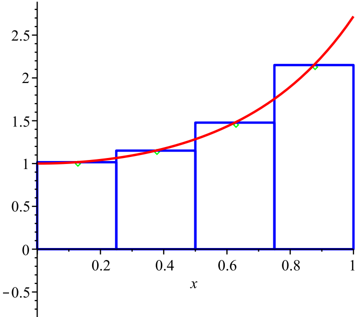
# Midpoint rule

Consider a function $f$ over an interval $[a, b]$ .

- $n$ number of subdivisions.
- $\Delta x = \frac{b-a}{n}$.
- $x_0 = a$, $x_i = a + i\Delta x$ for $i = 1$ to $n$. (note $x_n = b$).
- $m_i = \frac{x_i + x_{i-1}}{2}$ or $m_1 = a + \frac{\Delta x}{2}$, $m_i = m_{i+1} + \Delta x$ for $i > 1$

$$M_n = \sum_{i=1}^{n} f(m_i)\Delta x =$$

$$= f(m_1)\Delta x + f(m_2)\Delta x + \cdots + f(m_n)\Delta x$$



## Example

Approximate the area under the parabola $y = x^2 + 1$ in the interval $[0\,,1]$ using the midpoint rule with 4 subdivision.

## Example

If you use the midpoint rule with 4 subdivisions on $f(x) = x^2 - 5$ in the interval $[0, 3]$ which value do you get and what does this value approximate ?

# Properties of the definite integral

- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
- $\int_a^a f(x)\,dx = 0$
- $\int_a^b c\,dx = c(b-a)$
- $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

- $\int_a^b (f(x) + g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
- $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$
- If $f(x) \geq 0$ on $a \leq x \leq b$ then $\int_a^b f(x)\,dx \geq 0$

- If $f(x) \geq g(x)$ on $a \leq x \leq b$ then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$
- If $m \leq f(x) \leq M$ on $a \leq x \leq b$ then $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$

Estimate the distance traveled by a car in the time interval [0, 2] , given that the car 's velocity at time $t$ is $v(t) = 2\sqrt{t}$

Express the limit as a definite integral on [1, 5] :
$\lim_{n\to\infty} \sum_{i=1}^{n} (1 + (i\frac{4}{n}))^2 \, \frac{4}{n}$