

Lesson1

Read 4.9

Antiderivative of a function

Rectilinear motion, free fall

- ▶ web page : <https://sites.math.washington.edu/~ep2/classes/125/125.html>
- ▶ email : ep2@uw.edu
- ▶ Exam dates
- ▶ WebAssign

Things to review

- ▶ Elementary functions : x^n , $\frac{1}{x}$, \sqrt{x} , e^x $\ln x$, trig functions.

- ▶ Derivatives: formulas, rules , interpretation.

Function F Derivative F'

$$c \qquad 0$$

$$x^n \qquad nx^{n-1}$$

$$e^x \qquad e^x$$

$$\ln(x) \qquad \frac{1}{x}$$

$$\sin x \qquad \cos x$$

$$\cos x \qquad -\sin x$$

$$\tan(x) \qquad \sec^2 x$$

Function F	Derivative F'
$\arctan x$	$\frac{1}{1+x^2}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$\frac{-1}{\sqrt{1-x^2}}$
$\cotan x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec}(x)\cotan x$

In this class we study **antiderivatives**.

Given a function f (we think of $f = F'$) we want to find another function F such that $F' = f$

Def: A function F defined on an interval I is called an **antiderivative** of another function f defined on I if and only if $F'(x) = f(x)$ for all x in I .

Example

Find an antiderivative for $f(x) = 2x$

Function $f = F'$ Particular antiderivative F

$$x^n \quad n \neq -1$$

$$\frac{x^{n+1}}{n+1}$$

$$\frac{1}{x}$$

$$\ln |x|$$

$$e^x$$

$$e^x$$

$$\sin x$$

$$-\cos x$$

$$\cos x$$

$$\sin x$$

$$\frac{1}{1+x^2}$$

$$\arctan(x)$$

Function $f = F'$ Particular antiderivative F

$$\sec^2(x)$$

$$\tan(x)$$

$$\operatorname{cosec}^2(x)$$

$$-\cotan(x)$$

$$\sec(x)\tan(x)$$

$$\sec(x)$$

$$\frac{1}{\sqrt{1-x^2}}$$

$$\arcsin(x)$$

$$\frac{-1}{\sqrt{1-x^2}}$$

$$\arccos(x)$$

Does every function have an antiderivative?

If a function f has an antiderivative F , is it unique?

Differentiation rules

$$(cF)' = cF'$$

$$(F + G)' = F' + G'$$

Antidifferentiation rules

Assume $F' = f$ and $G' = g$

An antiderivative of cf is cF

An antiderivative of $(f + g)$ is $F + G$.

Find an antiderivative for

$$f(x) = 3\sqrt{x} + \frac{\cos x}{2} + \frac{1}{x^2}$$

then find ALL antiderivatives.

Challenge question

Find an antiderivative for $f(x) = |x - 1|$, then find ALL antiderivatives.

Application to rectilinear motion

- ▶ $s(t)$ position of a certain object, with respect to a given origin, at time t
- ▶ $v(t) = s'(t)$ velocity of the object at time t .
- ▶ $a(t) = v'(t)$ acceleration of the object at time t .

Free fall

Given that an object in free fall has a constant acceleration $g = -9.8 \text{ m/sec}^2$ ($=-32 \text{ feet/sec}^2$) find a general formula for the position of the object at time t .

A ball is dropped from a location 100 m above the ground. Find the distance of the ball above the ground level at time t .

How long does it take the ball to reach the ground?

With what velocity does it strike the ground?

A motorist is driving along a straight road with $v(t) = 30(t - t^2)$ km/h. After 1 hr it reaches town A. Find the distance of the motorist from town A after 2 hr.