1) $\int \sqrt{x} \cdot \ln x \, dx$

2) $\int \frac{\ln x}{x} dx$

3) $\int \frac{\ln x}{x^2} dx$

4) $\int \frac{1}{x(\ln x)^2} dx$

5) $\int x (\ln x)^2$

6) $\int \tan^3 x \, dx$

7) $\int_0^{\pi/4} \frac{\tan^3(x)}{\cos^2(x)} dx$

8) $\int \cos^3 x \, dx$

9) $\int \sin^2 x \, d$

10) $\int \sin^2 x \cdot \cos^3 x \, dx$

11) $\int \frac{e^x}{1+e^{2x}} dx$

12) $\int_0^{\sqrt{3}} x \tan^{-1} x \, dx$

13) $\int \frac{\tan^2 t \cdot \sec^2 t}{1+\tan t}$

14) $\int \tan^3 x \cdot \sin^2 x \, dx$

15) $\int \sin t \cdot \cos t \, e^{\cos t} \, dt$

16) $\int_0^{\pi/3} \tan^4(\theta) \, d\theta$

17) $\int (\sec x)^3 \tan x \, dx$

18) $\int x \sec x \tan x \, dx$

1) $\int \underset{v'}{\sqrt{x}}\ \underset{u}{\ln x}\ dx$ By parts

$v = \frac{2}{3} \cdot x^{3/2}$

$= \ln x \cdot \frac{2}{3} x^{3/2} - \frac{2}{3} \int \frac{1}{\cancel{x}} x^{\cancel{3}/2} dx$

$= \boxed{\frac{2}{3} \ln x \cdot x^{3/2} - \frac{4}{9} x^{3/2} + C}$

2) $\int \frac{\ln x}{x} dx$ by substitution

$u = \ln x$
$du = \frac{1}{x} dx$

$\int u \, du = \frac{u^2}{2} + C$

$\boxed{\frac{(\ln x)^2}{2} + C}$

3) $\int \frac{\ln x}{x^2} dx$ by parts

$v' = \frac{1}{x^2}$ $v = -\frac{1}{x}$, $u = \ln x$

$= -\frac{\ln x}{x} + \int \frac{1}{x^2} dx = \boxed{-\frac{\ln x}{x} - \frac{1}{x} + C}$

4) $\int \frac{1}{x(\ln x)^2}\,dx$ by substitution

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int \frac{1}{u^2}\,du = -\frac{1}{u} + C$

$\boxed{-\frac{1}{\ln x} + C}$

5) $\int \underset{v'}{x}\,\underset{u}{(\ln x)^2}\,dx$ by parts

$v = \frac{x^2}{2}$

$\frac{x^2}{2}(\ln x)^2 - \int \overset{v'}{\frac{x^{\cancel{2}}}{\cancel{2}}} \cdot \cancel{2}\,\overset{u}{\ln x} \cdot \frac{1}{\cancel{x}}\,dx$ by parts again

$v = \frac{x^2}{2}$

$= \frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \int \frac{1}{\cancel{x}} \cdot \frac{x^{\cancel{2}}}{2}\,dx =$

$\frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \frac{1}{4}x^2 + C$

6) $\int \tan^3 x \, dx$

I would like to have a "sec x" term as well

$\tan^3 x = \tan^2 x \tan x = (\sec^2 x - 1)\tan x$

$$\int \tan x \cdot \sec x \cdot \sec x - \tan x \, dx$$

$\int \tan x = \int \frac{\sin x}{\cos x} dx$ $\quad u = \cos x$

$\quad du = -\sin x \, dx$

$\int -\frac{du}{u} = -\ln|u|$

$-\ln|\cos x| + C$

$\int \tan x \cdot \sec x \cdot \sec x \, dx$ $\quad u = \sec x$

$\quad du = \tan x \sec x \, dx$

$\int u \, du = \frac{u^2}{2} + C$

$\frac{(\sec x)^2}{2} + C$

Therefore $\int \tan^3 x \, dx =$

$$\frac{(\sec x)^2}{2} + \ln|\cos x| + C$$

7) $\displaystyle\int_0^{\pi/4} \frac{\tan^3(x)}{\cos^2(x)}\,dx = \int_0^{\pi/4} \frac{\sin^3(x)}{\cos^5(x)}\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$$\int_{\cos 0}^{\cos(\pi/4)} -\frac{(1-u^2)}{u^5}\,du = \frac{u^{-4}}{4} - \frac{u^{-2}}{2}\Bigg|_1^{1/\sqrt{2}}$$

$$=\left(\frac{1}{4}\cdot 4 - \frac{1}{2}\,2\right) - \left(\frac{1}{4} - \frac{1}{2}\right) = \boxed{\frac{1}{4}}$$

8) $\displaystyle\int \cos^3 x\,dx = \int \cos^2 x \cdot \cos x\,dx$

$u = \sin x$ $\qquad \displaystyle\int (1-u^2)\,du = u - \frac{u^3}{3} + c$

$du = \cos x\,dx$

$$\boxed{\sin x - \frac{\sin^3 x}{3} + c}$$

9) $\displaystyle\int \sin^2 x\,dx$ Even power

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\displaystyle\int \frac{1}{2}\,dx - \int \frac{1}{2}\cos(2x) = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$

10) $\int \sin^2 x \cos^3 x \, dx$

$= \int \sin^2 x \cos^2 x \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$\int u^2 (1-u^2) \, du = \int u^2 - u^4 \, du$

$= \frac{u^3}{3} - \frac{u^5}{5} + C$

$\boxed{\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C}$

11) $\int \frac{e^x}{1+e^{2x}} \, dx$

$u = e^x$

$du = e^x \, dx$

$\int \frac{du}{1+u^2} = \arctan u + C$

$\arctan(e^x) + C$

12) $\displaystyle\int_0^{\sqrt{3}} \underset{v'}{x} \cdot \underset{u}{\tan^{-1} x}\, dx$ by parts

$v = \dfrac{x^2}{2}$ $\qquad \dfrac{x^2}{2} \tan^{-1} x \Big|_0^{\sqrt{3}} - \displaystyle\int_0^{\sqrt{3}} \frac{x^2}{2} \frac{1}{1+x^2}\, dx$

$= \dfrac{3}{2} \tan^{-1} \sqrt{3} - \dfrac{1}{2} \displaystyle\int_0^{\sqrt{3}} \frac{x^2}{1+x^2}\, dx$

$\dfrac{3}{2} \cdot \dfrac{\pi}{3} - \dfrac{1}{2} \displaystyle\int_0^{\sqrt{3}} \frac{x^2+1-1}{1+x^2}\, dx = \frac{\pi}{2} - \frac{1}{2}\left(x - \arctan x\right)\Big|_0^{\sqrt{3}}$

$\dfrac{\pi}{2} - \dfrac{\sqrt{3}}{2} + \dfrac{1}{2}\dfrac{\pi}{3} = \boxed{\dfrac{2}{3}\pi - \dfrac{\sqrt{3}}{2}}$

13) $\displaystyle\int \frac{\tan^2 t \cdot \sec^2 t}{1+\tan t}\, dt$ $\qquad u = \tan t$

$du = \sec^2 t\, dt$

$\displaystyle\int \frac{u^2}{1+u}\, du = \int \frac{u^2-1+1}{1+u}\, du = \int \frac{(u+1)(u-1)}{1+u} + \frac{1}{1+u}\, du$

$= \dfrac{u^2}{2} - u + \ln|u+1| + C;$ $\boxed{\dfrac{\tan^2 t}{2} - \tan t - \ln|\tan t + 1| + C}$

14) $\int \tan^3 x \sin^2 x \, dx$

$\int \frac{\sin^4 x \quad \sin x}{\cos^3 x} dx$ $\qquad u = \cos x$

$du = -\sin x \, dx$

$\int -\frac{(1-u^2)^2}{u^3} du = \int -\frac{(1-2u^2+u^4)}{u^3} du$

$= \int -\frac{1}{u^3} + \frac{2}{u} - u \, du = \frac{1}{2} u^{-2} + 2\ln|u| - \frac{u^2}{2} + C$

$$\boxed{\frac{1}{2\cos^2 x} + 2\ln|\cos x| - \frac{\cos^2 x}{2} + C}$$

15) $\int \sin t \cdot \cos t \, e^{\cos t} dt$ $\qquad u = \cos t$

$du = -\sin t \, dt$

$-\int \underset{a}{u} \underset{b'}{e^u} du = -ue^u + \int e^u du = -ue^u + e^u + C$

$b = e^u$ $\qquad -\cos t \, e^{\cos t} + e^{\cos t} + C$

16) $\int_0^{\pi/3} \tan^4(\theta) \, d\theta = \int_0^{\pi/3} \tan^2\theta (\sec^2\theta - 1) \, d\theta$

$= \int_0^{\pi/3} \tan^2\theta \sec^2\theta \, d\theta - \int_0^{\pi/3} \sec^2\theta - 1 \, d\theta$ $\qquad u = \tan\theta$

$du = \sec^2\theta \, d\theta$

$= \int_{\tan}^{\tan \pi/3} u^2 du - \tan\theta \Big|_0^{\pi/3} + u\Big|_0^{\pi/3} = \frac{u^3}{3}\Big|_0^{\sqrt{3}} - \sqrt{3} + \frac{\pi}{3}$

$= \boxed{\pi/3}$

$=$

17) $\int \sec^5 x \tan x \, dx$

$u = \sec x$

$du = \sec x \tan x \, dx$

$\int u^4 \, du = \frac{u^5}{5} + C$

$$\boxed{\frac{(\sec x)^5}{5} + C}$$

18) $\int \underbrace{x}_{u} \cdot \underbrace{\sec x \tan x}_{v'} \, dx$

$v = \sec x$

$x \sec x - \int \sec x \, dx =$

$x \sec x - \ln|\sec x + \tan x| + C$