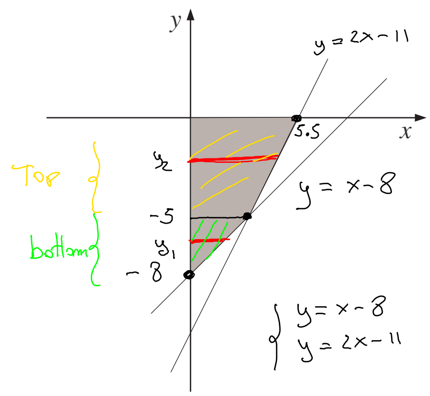
6. (10 points) The shaded region shown below is bounded by the y-axis, the x-axis, the line $y = x - 8$, and the line $y = 2x - 11$. It is revolved about the y-axis to create a tank, with units in meters. ~~The tank is filled with a liquid with density 800 kg~~/m^3. Express ~~the work required~~ **the volume of the tank** to ~~pump all of the liquid out over the top of the ta~~nk in terms of integrals, but do not evaluate these integrals.



$\begin{cases} y = x - 8 \\ y = 2x - 11 \end{cases}$ $\begin{cases} 2x - 11 = x - 8 \\ y = -5 \end{cases}$ $x = 3$

V slice is $\pi r^2 \Delta y$ $r = x$

- $x = y + 8$ at the bottom of the conteiner $(-8 < y < -5)$
- $x = \frac{y+11}{2}$ at the top of the container $(-5 < y < 0)$

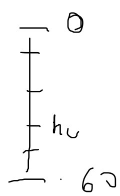
$$W = \int_{-8}^{-5} \pi \cdot (y+8)^2 \, dy + \int_{-5}^{0} \pi \cdot \left(\frac{y+11}{2}\right)^2 dy$$

A 60 feet chain that weights 30 lb is dangling from the roof of the building. How much work is needed to pull the chain up to the top of the building?.

$$\rho = \frac{30}{60} = \frac{1}{2} \quad lb/ft$$

$$\int_0^{60} \frac{1}{2} x \, dx$$

Attached to the chain is a bucket full of water that weights 30lb but looses water at a rate of 0.25 lb per second once you start pulling the chain up. Assume that you pull the chain+ bucket up at a rate of 2 feet per second. Find the work needed to pull chain and bucket to the top of the building.



$$\Delta W = \text{weight at } h_i \cdot \Delta h$$

$$\text{weight at time } t_i = 30 - 0.25 t_i \qquad t_i = \frac{h_i}{v} = \frac{h_i}{2}$$

$$\text{weight at } h_i = 30 - \frac{0.25}{2} h_i$$

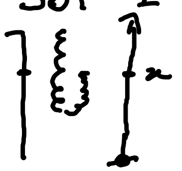
$$W = \int_0^{60} \left(30 - \frac{0.25}{2} h\right) dh$$

A robot weighting 100 lb is attached to a cable weighting 0.8 lb/foot and then lowered into a 30 feet well. The robot gets out of the well by climbing up the cable with one of the ends still attached to it. Calculate the work done by the robot in climbing out of the well.

Work to lift robot : $100 \cdot 30 = 3000$ lb ft

Work to lift cable

Sol 1



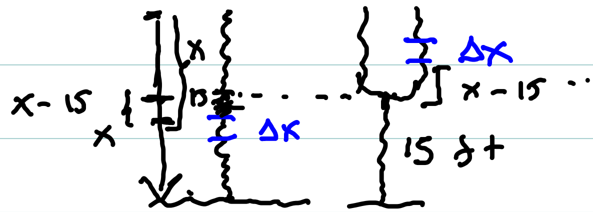
origin at bottom of well

- work done when robot moves up from height x to height $x + \Delta x$ is:
- $W_c = \underbrace{0.8 \cdot \frac{x}{2}}_{\text{weight of chain robot is pulling up}} \underbrace{\Delta x}_{\text{distance}}$

total work for cable $\displaystyle\int_0^{30} 0.8 \cdot \frac{x}{2}\, dx = 0.4 \cdot \frac{x^2}{2} \Big|_0^{30} = 180$ lb$\cdot$ft

Total work 3180 lb$\cdot$ft

Sol 3



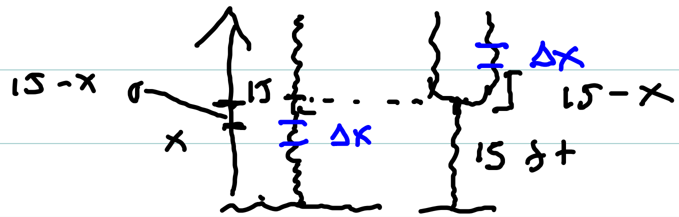
work done on small segment Δx of cable at distance x from top of the well

$$W_c = \underbrace{2(x-15)}_{\text{distance}}\ \underbrace{0.8\,\Delta x}_{\text{weight of small segment}}$$

total work for cable $\displaystyle\int_{15}^{30} 2\cdot(x-15)\,0.8\,dx = 0.8\cdot(x-15)^2\Big|_{15}^{30}$

$= 0.8\cdot 15^2 = 180$

Sol 2



work done to pull up small segment Δx of cable at distance x from bottom of the well

$$W_L = \underbrace{2(15-x)}_{\text{distance}} \underbrace{0.8\,\Delta x}_{\text{weight of small segment}}$$

total work for cable $\int_0^{15} 2\cdot(15-x)\,0.8\,dx = -0.8\,(15-x)^2\Big|_0^{15}$

$= 0.8 \cdot 15^2 = 180$