Math 125 Winter 2019

PLEASE WRITE CLEARLY.

NAME : ....

Student ID ....

- IMPORTANT: write your solution BELOW the problem and DO NOT write within 1 cm of the edge. If you run out of space, continue your work in the back of the previous page and indicate clearly on the problem page that you have done so.
- Your exam will be scanned, so make sure your writing is clear and dark enough to appear on the scan
- Write your NAME (first, last) on top of every even page, except this one.
- Your exam contains 5 problems. The entire exam is worth 50 points. This exam has 6 pages. Please make sure that your exam is complete.
- You have 80 minutes to complete this exam.
- This exam is closed book. You may use one $8\frac{1}{2}$" $\times$ 11" sheet of notes (both sides). Do not share notes. The only calculator allowed is the TI 30x IIS.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Unless the problem asks for an exact answer, you can give a decimal approximation, rounding off to three decimal digits.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- Raise your hand if you have a question.

1. Calculate the following integrals. Leave your answers in exact form simplifying as much as possible.

$U = x^2 + 2$
$dU = 2x\,dx$

(a) $\displaystyle\int \frac{1}{\sqrt{2x+5}} + x^3\sqrt{x^2+2}\,dx$ $= \int (2x+5)^{-1/2} + \int x^3 \sqrt{x^2+2}\,dx$

$\int (2x+5)^{-1/2} dx = (2x+5)^{1/2}$

$\int \frac{1}{2}(u-2)\sqrt{u}\,du = \frac{1}{2}\frac{u^{5/2}}{\frac{5}{2}} - \frac{u^{3/2}}{\frac{3}{2}} + C$

$\sqrt{2x+5} + \frac{(x^2+2)^{5/2}}{5} - \frac{2}{3}(x^2+2)^{3/2} + C$

or

$\sqrt{2x+5} + \frac{(x^2+2)^2\sqrt{x^2+2}}{5} - \frac{2}{3}(x^2+2)\sqrt{x^2+2} + C$

(b) $\displaystyle\int_1^2 \frac{\sin(3\ln x+1)}{x}\,dx$

$U = 3\ln x + 1$
$dU = \frac{3}{x}\,dx$

$\displaystyle\int_{3\ln 1+1}^{3\ln 2+1} \sin(u)\,\frac{du}{3} = -\frac{\cos u}{3}\Big|_1^{3\ln 2+1} = \frac{1}{3}\left(\cos 1 - \cos(3\ln 2+1)\right)$

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2. Consider $\int_0^1 \sin x^5\, dx$.

(a) Write a formula for the right hand sum approximation with $n = 3$ subdivisions,(R_3) for the integral above.

$\Delta x = \frac{1}{3}$, $x_0 = 0$, $x_1 = \frac{1}{3}$, $x_2 = \frac{2}{3}$, $x_3 = 1$

$R_3 = \left(\sin\left(\frac{1}{3^5}\right) + \sin\left(\left(\frac{2}{3}\right)^5\right) + \sin(1) \right) \cdot \frac{1}{3}$

(b) Evaluate (R_3) to approximate the integral above.

$R_3 = \left(\sin\left(\frac{1}{3^5}\right) + \sin\left(\left(\frac{2}{3}\right)^5\right) + \sin(1) \right) \cdot \frac{1}{3} \approx 0.326$

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3. The acceleration of an object moving along a straight line, with initial velocity $v_0 = 2$ feet/sec is given by $a(t) = 2t - 3$ feet/sec^2.

a) Calculate the displacement of the object in the time interval $[0, 2]$

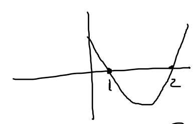
$v(t) = t^2 - 3t + 2$

$$\text{displacement} = \int_0^2 t^2 - 3t + 2\,dt = \frac{t^3}{3} - 3\frac{t^2}{2} + 2t \Big|_0^2$$

$$= \frac{8}{3} - 6 + 4 = \frac{8-6}{3} = \frac{2}{3} \text{ feet}$$

b) Calculate the distance traveled by the object in the time interval $[0, 2]$

$$\text{distance} = \int_0^2 |t^2 - 3t + 2|\,dt = \int_0^1 t^2 - 3t + 2$$



$$+ \int_1^2 -t^2 + 3t - 2 = \frac{t^3}{3} - 3\frac{t^2}{2} + 2t \Big|_0^1 + -\frac{t^3}{3} + 3\frac{t^2}{2} - 2t \Big|_1^2 =$$

$$= \frac{1}{3} - \frac{3}{2} + 2 - \frac{8}{3} + 6 - 4 + \frac{1}{3} - \frac{3}{2} + 2 = \frac{2}{3} - 3 + 4 - \frac{2}{3} = 1 \text{ foot}$$

c) The object reaches a point A on the line at time $t = 1$. What is the object 's distance from A at time $t = 2$?

Let A be the origin then

$s(t) = \underline{t^3} - 3\frac{t^2}{2} + 2t + S_0$ and $S(1) = 0$ so

$\frac{1}{3} - \frac{3}{2} + 2 + S_0 = 0$ $\quad S_0 = \frac{9 - 2 - 12}{6} = -\frac{5}{6}$

$$|S(2)| = \left|\frac{8}{3} - 3\cdot\frac{4}{2} + 2\cdot 2 - \frac{5}{6}\right| = \left|\frac{11}{6} - \frac{12}{6}\right| = \frac{1}{6} \text{ feet}$$

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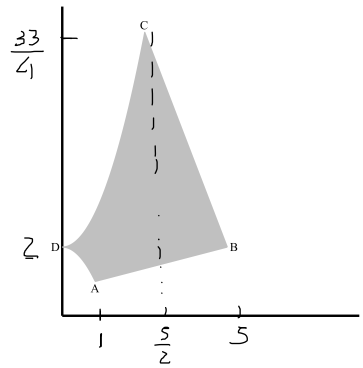
4. Calculate the area of the shaded region below. AB is a segment of the straight line $y = \frac{x}{4} + \frac{3}{4}$, BC is a segment of the straight line $y = -\frac{5}{2}x + \frac{29}{2}$, CD is along the curve $y = x^2 + 2$ and DA is along the curve $y = -x^2 + 2$. $A = (1, 1)$, $B = (5, 2)$, and $C = (\frac{5}{2}, \frac{33}{4})$.



$x^2 + 2 = -x^2 + 2$

$x = 0$

$D\ (0, 2)$

$$\int_1^2 4\left(y - \frac{3}{4}\right) - \sqrt{2-y}\ dy + \int_2^{33/4} -\frac{2}{5}\left(y - \frac{29}{2}\right) - \sqrt{y-2}\ dy =$$

$$\frac{7}{3} + \frac{625}{48} = \frac{737}{48} \approx 15.354$$

or

$$\int_0^1 (x^2+2) - (-x^2+2)\,dx + \int_1^{5/2} (x^2+2) - \left(\frac{x}{4} + \frac{3}{4}\right) dx + \int_{5/2}^{5} \left(-\frac{5}{2}x + \frac{29}{2}\right) - \left(\frac{x}{4} + \frac{3}{4}\right) dx$$

$$\frac{2}{3} + \frac{195}{32} + \frac{275}{32}$$

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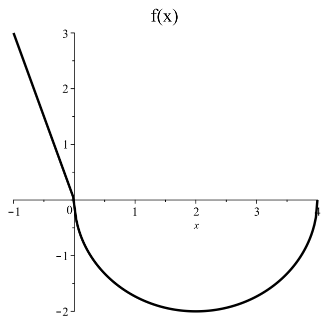
5. The graph of $f(x)$ below consists of a straight line and a semicircle. Give exact answers.



(a) Calculate $\left|\int_{-1}^{4} f(x)\,dx\right|$

$$\left|\frac{1}{2}\cdot 1\cdot 3-\frac{1}{2}\pi\cdot 2^2\right| = 2\pi-\frac{3}{2}$$

(b) Calculate $\int_{-1}^{4} |f(x)|\,dx$ $= 2\pi + \frac{3}{2}$

(c) Let $h(x)=\int_0^{2x} f(t)\,dt$. Compute $h'(1)$

$$h'(x) = f(2x)\cdot 2$$
$$h'(1) = f(2)\cdot 2 = -4$$

(d) Let $g(x)=\int_0^{x} f(t)\,dt$. Is g concave up or down at $x=-\frac{1}{2}$? Justify your answer.

$f = g'$ is decreasing at $x=-\frac{1}{2}$

so concave down

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1. Calculate the following integrals. Leave your answers in exact form simplifying as much as possible.

(a) $\int x^5\sqrt{x^2+1}+\sqrt{3x+2}\,dx$

$U = x^2+1$

$dU = 2x\,dx$

$\int (U-1)^2 \sqrt{U}\ \frac{dU}{2} = \int \frac{U^2\sqrt{U}}{2} - U\sqrt{U} + \frac{\sqrt{U}}{2}dU = \frac{1}{2}\frac{U^{7/2}}{\frac{7}{2}} - \frac{U^{5/2}}{\frac{5}{2}} + \frac{1}{2}\frac{U^{3/2}}{\frac{3}{2}} + C$

$\int \sqrt{3x+2}\ dx = \frac{1}{3}\frac{(3x+2)^{3/2}}{3/2} + C$

$\frac{1}{7}(x^2+1)^{7/2} - \frac{2}{5}(x^2+1)^{5/2} + \frac{1}{3}(x^2+1)^{3/2} + \frac{2}{9}(3x+2)^{3/2} + C$

or

$\frac{1}{7}(x^2+1)^3\sqrt{x^2+1} - \frac{2}{5}(x^2+1)^2\sqrt{x^2+1} + \frac{1}{3}(x^2+1)\sqrt{x^2+1} + \frac{2}{9}(3x+2)\sqrt{3x+2} + C$

(b) $\int_0^1 e^x\cos(2e^x+7)\,dx$

$U = 2e^x+7$

$dU = 2e^x\,dx$

$\int_{2e^0+7}^{2e^1+7} \cos(U)\,\frac{dU}{2} = \frac{1}{2}\sin U\Big|_9^{2e+7} = \frac{1}{2}\sin(2e+7) - \frac{1}{2}\sin 9$

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2. Consider $\int_0^1 e^{x^5}\,dx$. .

(a) Write a formula for the left hand sum approximation with $n = 3$ subdivisions,(L_3) for the integral above.

$\Delta x = \frac{1}{3}$ $\quad x_0 = 0, \; x_1 = \frac{1}{3}, \; x_2 = \frac{2}{3}, \; x_3 = 1$

$L_3 = \left(e^{0^5} + e^{(1/3)^5} + e^{(2/3)^5}\right) \cdot \frac{1}{3} \approx 1.048$

(b) Evaluate (L_3) to approximate the integral above.

$L_3 = \left(e^{0^5} + e^{(1/3)^5} + e^{(2/3)^5}\right) \cdot \frac{1}{3} \approx 1.048$

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3. The acceleration of an object moving along a straight line, with initial velocity $v_0 = 4$ feet/sec , is given by $a(t) = 2t - 5$ feet/sec ^{2}.

a) Calculate the displacement of the object in the time interval $[0, 3]$

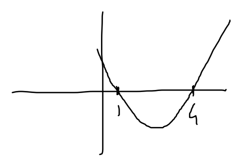
$V(t) = t^2 - 5t + 4$

displacement $\int_0^3 t^2 - 5t + 4\,dt = \frac{t^3}{3} - \frac{5}{2}t^2 + 4t \Big|_0^3$

$= 9 - \frac{5}{2}\cdot 9 + 12 = \frac{42-45}{2} = -\frac{3}{2}$ feet

b) Calculate the distance traveled by the object in the time interval $[0, 3]$

$\int_0^3 |t^2 - 5t + 4|\,dt = \int_0^1 t^2 - 5t + 4\,dt$



$+ \int_1^3 -t^2 + 5t - 4\,dt = \frac{t^3}{3} - \frac{5t^2}{2} + 4t\Big|_0^1$

$+ -\frac{t^3}{3} + \frac{5t^2}{2} - 4t\Big|_1^3 = \frac{1}{3} - \frac{5}{2} + 4 + (-9 + 5\cdot\frac{9}{2} - 12) + \frac{1}{3} - \frac{5}{2} + 4$

$= \frac{2}{3} - 5 + 8 + \frac{3}{2} = \frac{4 + 18 + 9}{6} = \frac{31}{6}$ feet

c) The object reaches a point A on the line at time $t = 2$. What is the object 's distance from A at time $t = 3$?

Let A be the origin

$s(t) = \frac{t^3}{3} - \frac{5t^2}{2} + 4t + s_0$

$s(2) = 0$ so $\frac{8}{3} - 10 + 8 + s_0 \Rightarrow s_0 = -\frac{2}{3}$

$|s(3)| = |9 - \frac{45}{2} + 12 - \frac{2}{3}| = |\frac{126 - 135 - 4}{6}| = \frac{13}{6}$ feet

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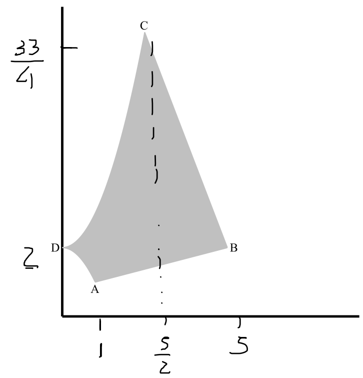
4. Calculate the area of the shaded region below. AB is a segment of the straight line $y = \frac{x}{4} + \frac{3}{4}$, BC is a segment of the straight line $y = -\frac{5}{2}x + \frac{29}{2}$, CD is along the curve $y = x^2 + 2$ and DA is along the curve $y = -x^2 + 2$. $A = (1, 1)$, $B = (5, 2)$ and $C = (\frac{5}{2}, \frac{33}{4})$.



$x^2 + 2 = -x^2 + 2$

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$D\ (0, 2)$

$$\int_1^2 4\left(y - \frac{3}{4}\right) - \sqrt{2-y}\ dy + \int_2^{33/4} -\frac{2}{5}\left(y - \frac{29}{2}\right) - \sqrt{y-2}\ dy =$$

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$$\frac{2}{3} + \frac{195}{32} + \frac{275}{32}$$

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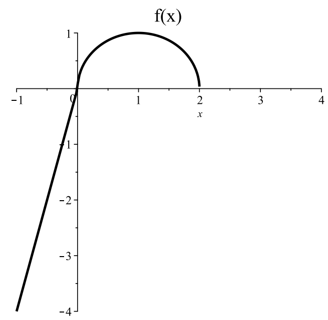
5. The graph of $f(x)$ below consists of a straight line and a semicircle. Give exact answers.



(a) Calculate $\left| \int_{-1}^{2} f(x)\,dx \right|$ $= \left| \frac{-1}{2} \cdot 1 \cdot 4 + \frac{\pi}{2} \right| = 2 - \frac{\pi}{2}$

(b) Calculate $\int_{-1}^{2} |f(x)|\,dx$ $= 2 + \frac{\pi}{2}$

(c) Let $h(x) = \int_0^{4x} f(t)\,dt$. Compute $h'(\frac{1}{4})$

$h'(x) = f(4x) \cdot 4$

$h'(\frac{1}{4}) = f(1) \cdot 4 = 4$

(d) Let $g(x) = \int_0^{x} f(t)\,dt$. Is g concave up or down at $x = -\frac{1}{2}$? Justify your answer.

$f = g'$ is increasing at $x = -\frac{1}{2}$ so concave up

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