Math 125 Section D (Pezzoli)
Fall 2017
Midterm #1 (60 points)

Name ______________

TA: ______________

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Section: ______________

- Your exam contains 4 problems. The entire exam is worth 60 points.
- This exam is closed book. You may use one $8\frac{1}{2}$" $\times 11$" sheet of notes (both sides). Do not share notes.
- The only calculator allowed is the TI 30x IIS.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. Show your work in evaluating integrals, even if they are on your note sheet.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- Give your answer in exact form means write $\sin(2) - \pi$ instead of -2.23. If you are asked to give a decimal approximation, use two decimal digits in your final answer.
- This exam has 5 pages,including this cover sheet. Please make sure that your exam is complete.

Problem #1(20 pts) ______

Problem #2(10 pts) ______

Problem #3(15 pts) ______

Problem #4(15 pts) ______

TOTAL (60 pts) ______

1. Calculate the following integrals. Give your answers in exact form.

(a) $\int_0^1 x \cdot e^{x^2} \cdot \sin\left(e^{x^2}\right) dx$

$U = e^{x^2}$

$dU = 2x e^{x^2} dx$

$\int_{e^0}^{e^1} \frac{1}{2} \sin U \, dU = \frac{1}{2}(-\cos U)\Big|_1^e = \boxed{\frac{1}{2}(\cos 1 - \cos e)}$

(b) $\int x^3\sqrt{x^2-2}\,dx$

$U = x^2 - 2$, $x^2 = U + 2$

$dU = 2x\,dx$

$\int \frac{1}{2}(U+2)\sqrt{U}\,dU$

$\int \frac{1}{2}(U+2)\sqrt{U}\,dU = \int \frac{1}{2} U^{3/2} + U^{1/2} = \frac{1}{5} U^{\frac{5}{2}} + \frac{2}{3} U^{\frac{3}{2}} + C$

$\boxed{\frac{1}{5}(x^2-2)^{5/2} + \frac{2}{3}(x^2-2)^{3/2} + C}$

(problem 1 continued)

(c) $\displaystyle\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \frac{1}{\tan(2\theta)} d\theta \quad = \int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \frac{1}{\frac{\sin(2\theta)}{\cos(2\theta)}} \quad = \int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \frac{\cos(2\theta)}{\sin(2\theta)}\, d\theta$

$u = \sin(2\theta)$

$du = 2\cos(2\theta)\, d\theta$

$$\int_{\sin(\frac{\pi}{6})}^{\sin(\frac{\pi}{3})} \frac{1}{2}\,\frac{du}{u} = \frac{1}{2}\ln|u| \Big|_{1/2}^{\frac{\sqrt{3}}{2}} = \boxed{\frac{1}{2}\left(\ln\frac{\sqrt{3}}{2} - \ln\frac{1}{2}\right)} \quad \text{or}$$

$$\frac{\ln 3}{8} + \frac{\ln 2}{2}$$

2. Given $f(x) = \displaystyle\int_{\cos(x)}^{\sin(x)} e^{t^2}\, dt$, calculate $f'(0)$. Is f increasing or decreasing at $x = 0$?

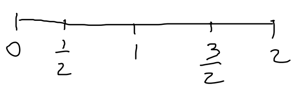
$$\int_{\cos(x)}^{c} e^{t^2}\, dt + \int_{c}^{\sin x} e^{t^2}\, dt = -\int_{c}^{\cos(x)} e^{t^2} dt + \int_{c}^{\sin x} e^{t^2} dt$$

$f'(x) = -e^{\cos^2(x)} \cdot (-\sin x) + e^{\sin^2 x} \cos(x).$

$f'(0) = 1 > 0$ so increasing

3. John leaves his house at time $t = 0$ and travels along a straight road. His velocity, in km/hr is given by the function $v_1(t) = 60\sin(t^2)t$.

   a) Write a Riemann sum with $n = 4$ subdivisions, taking the sample points to be right points (i.e. a right hand sum) that approximates the <u>distance</u> traveled by the car during the first 2 hours of John 's trip . Evaluate the sum giving a decimal approximation of your final answer.



$$\Delta x = \frac{2}{4} = \frac{1}{2}$$

$$d = \frac{1}{2}\left(\left|\frac{60}{2}\sin\left(\frac{1}{4}\right)\right| + \left|60\sin(1)\right| + \left|60\cdot\frac{3}{2}\sin\left(\frac{9}{4}\right)\right| + \left|60\cdot 2\cdot \sin 4\right| \right)$$

$$\approx 109.38 \text{ km}$$

   b) John reaches his final destination after 3 hours. How far is this destination from John's house ? Give a decimal approximation.

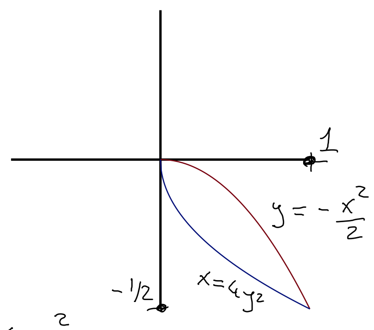
$$S(3) - S(0) = \int_0^3 60t \sin(t^2)\,dt$$

$u = t^2$

$du = 2t\,dt$

$$30\int_0^9 \sin(u)\,du = -30\cos u \Big|_0^9 = 30 - 30\cos 9 = 57.33 \text{ km}$$

4. Calculate the area of the region in the fourth quadrant bounded by the curves $x = 4y^2$ and $y = -\frac{x^2}{2}$.



Find intersections : $\begin{cases} x = 4y^2 \\ y = -\frac{1}{2} \cdot 16 y^4 \end{cases}$

$\begin{cases} -\frac{1}{8} y = y^4 , \ y = 0, -\frac{1}{2} \\ x = 0, 1 \end{cases}$

To integrate in dx solve $x = 4y^2$ for y : $y = \pm\sqrt{\frac{x}{4}}$

in fourth quadrant $y \leq 0$ so $y = -\frac{\sqrt{x}}{2}$

$$\int_0^1 -\frac{x^2}{2} + \frac{\sqrt{x}}{2} \, dx = -\frac{1}{6} x^3 + \frac{1}{3} x^{3/2} \Big|_0^1 = \frac{1}{3} - \frac{1}{6} = \frac{1}{6} \approx 0.17$$

OR To integrate in dy solve $y = -\frac{x^2}{2}$ for x : $x^2 = -2y$

$x = \pm\sqrt{-2y}$ in fourth quadrent $x \geq 0$ so $x = \sqrt{-2y}$

$$\int_{-\frac{1}{2}}^{0} \sqrt{-2y} - 4y^2 \, dy = -\frac{1}{3}(-2y)^{3/2} - \frac{4}{3} y^3 \Big|_{-\frac{1}{2}}^{0} = \frac{1}{3} - \frac{1}{6} = \frac{1}{6} \approx 0.17$$