

Lesson 8

Read 2.7 and 2.8

$f'(x_0)$ for multipart functions

The derivative function

Is f differentiable at $x_0 = 0$ if

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Is $f(x) = |x|$ differentiable at $x_0 = 0$?

Recall

- ▶ $f'(x_0)$ is the slope of the tangent line to f at x_0 , sometimes we say the slope of f at x_0 .
- ▶ $f'(x_0)$ tells us how f is changing at x_0 , increasing a lot (when $f'(x_0)$ is a big positive number), increasing a little (when $f'(x_0)$ is a small positive number), staying constant (when $f'(x_0) = 0$), decreasing a lot (when $f'(x_0)$ is a big negative number), decreasing a little (when $f'(x_0)$ is a small negative number)
- ▶ $f'(x_0)$ is the rate of change of f at x_0

2.7, 2.8 The derivative function

Given $f(x)$ we can calculate $f'(x_0)$ for different values of x_0

Example

If $f(x) = 2x$ calculate $f'(x_0)$ when $x_0 = 1, 2, 3$.

Find a general formula for $f'(x)$

Given $f(x) = \sqrt{x+2}$ calculate $f'(x)$

Differentiable functions

A function is called differentiable if $f'(x_0)$ exists for every x_0 in the domain of f

Second derivatives

The second derivative of f at x_0 is the derivative of the function $f'(x)$ at $x = x_0$, in other words is $(f')'(x_0)$. We will denote it by $f''(x_0)$ or $\frac{d^2f}{dx^2}(x_0)$

Example

Calculate $f''(x)$, if $f(x) = 2x$

Note

The second derivative of the position function $s(t)$ gives you the acceleration $a(t)$ of the moving object.

Higher order derivatives

We can also take the derivative of the second derivative of a function f , and call it the third derivative of f , in formulas $(f'')'(x) = f'''(x)$, and so on we can define 4th, 5th, ..., nth derivatives

Relations between f' , f

f'	f	f''
positive	increasing	-
negative	decreasing	-
increasing	concave up	positive
decreasing	concave down	negative