Lesson 7

Read 2.5 and 2.7

More on continuity

The derivative of f at x_0

Which values of m, if any, make f a continuous function ?

$$f(x) = \begin{cases} mx + 4 & \text{if } x \leq 1 \\ 3 - x^2 & \text{if } x > 1 \end{cases}$$

A function f is $\boxed{\text{continuous}}$ at x_0 from the right if:

- f is defined at x_0 and
- $\lim_{x \to x_0^+} f(x) = f(x_0)$
- Intuitive graphical explanation is that we don 't see any holes or gaps in the graph of f at x_0 if we don t look to the left of x_0

A function f is $\boxed{\text{continuous}}$ at x_0 from the left if:

- f is defined at x_0 and
- $\lim_{x \to x_0^-} f(x) = f(x_0)$
- Intuitive graphical explanation is that we don 't see any holes or gaps in the graph of f at x_0 if we don t look to the right of x_0

2.7 . Derivatives

Recall from lesson 3 that we often consider
$\lim_{x_2 \to x_1} \frac{f(x_2)-f(x_1)}{x_2-x_1} = \lim_{h \to 0} \frac{f(x_1+h)-f(x_1)}{h}$
This value (when it exists) is called the $\boxed{\text{derivative}}$ of f at x_1 and denoted by $f'(x_1)$ or $\frac{df}{dx}(x_1)$. When the limit exists we say that f is $\boxed{\text{differentiable at } x_0}$.
What are the units of $f'(x_0)$?

Also recall that $f'(x_1)$ is a number that is :

1. The slope of the tangent to the curve $y = f(x)$ at $P = (x_1, f(x_1))$.
2. The (instantaneous) rate of change of f at $t = x_1$.

The number of bacteria present in a Petri dish at time t, in hours, is given by the function $y = f(t)$

- What are the units of $f'(5)$?
- What is the meaning of f'(5) ?
- What does $f'(5) = 2000$ tell you ?
- What does $f'(6) = 0$ tell you ?
- What does the sign of $f'(t)$ tell you ?

Let $f(x) = x^2 + 2$. Using the definition of derivative calculate $f'(1)$.

Graphical estimate

Tangent line equation

The line tangent to $y = f(x)$ at $P = (x_0, f(x_0))$ is

$$\boxed{y = f(x_0) + f'(x_0)(x - x_0)}$$

Note

If f is differentiable at x_0 then it is continuous at x_0

A function that is continuous, but not differentiable at x_0

We can easily tell that f is not differentiable at x_0 if

- f is not continuous at x_0.
- The graph of f had a sharp corner at x_0.
- The graph of f is vertical at x_0.

Win 09 midterm (Nichifor)