Lesson 6

Read 2.3 and 2.6 and 2.5

Limits of functions defined by cases

Continuity

Limits of certain fractions

The idea behind computing limits of rational functions is to "'disregard smaller powers"' of x.
We can extendend this idea to more general fractions : if we want to compute $\lim_{x\to\infty} \frac{f(x)+g(x)}{h(x)+k(x)}$ we can try to "'disregard the smaller functions"' on top and bottom keeping in mind that "bigger functions win"' and that :

$$\text{bounded } < \ln x < \sqrt{x} < x^n < e^x$$

Example

Compute $\lim_{x\to\infty} \frac{x^{10}+1}{x^2+e^x}$

Limits of functions defined by cases

How to compute $\lim_{x\to x_0} f(x)$, where

$$f(x) = \begin{cases} g(x) & \text{if } x \le a \\ h(x) & \text{if } x > a \end{cases}$$

1. If $x_0 < a$ then $\lim_{x\to x_0} f(x) = \lim_{x\to x_0} g(x)$
2. If $x_0 > a$ then $\lim_{x\to x_0} f(x) = \lim_{x\to x_0} h(x)$
3. If If $x_0 = a$ then you need to compute the limits from the left and right separately and $\lim_{x\to x_0^-} f(x) = \lim_{x\to x_0^-} g(x)$, $\lim_{x\to x_0^+} f(x) = \lim_{x\to x_0^+} h(x)$

Example

Compute $\lim_{x\to 2} f(x)$, where

$$f(x) = \begin{cases} x+1 & \text{if } x \leq 2 \\ 3-x^2 & \text{if } x > 2 \end{cases}$$

Example

Compute $\lim_{x \to 2} f(x)$, where

$$f(x) = \begin{cases} x - 3 & \text{if } x \leq 2 \\ 3 - x^2 & \text{if } x > 2 \end{cases}$$

Example

Compute $\lim_{x \to 2} f(x)$, where

$$f(x) = \begin{cases} x^2 + 5 & \text{if } x \neq 2 \\ 6 & \text{if } x = 2 \end{cases}$$

2.5 Continuity

Recall :a function f is $\boxed{\text{continuous}}$ at a point x_0 if:

- f is defined at x_0 and
- $\lim_{x \to x_0} f(x) = f(x_0)$
- Intuitive graphical explanation is that f is continuous at x_0 if f is defined at x_0 and the graph of f has no holes or gaps at x_0

A function is continuous on a set S if it is continuous for every x_0 in S.

A function is continuous if it is continuous on its domain.

Removable discontinuity

A function f has a removable discontiuity at x_0 if it has a hole at x_0

Jump discontinuity

A function f has a jump discontiuity at x_0 if it has a finite gap at x_0

Infinite discontinuity

A function f has a infinite discontinuity at x_0 if it has an infinite gap at x_0

Recall all elementary functions are continuous. Any function created by adding,subtracting, multiplying, dividing elementary functions is continuous.

Example

Find all intervals on which $f(x) = \frac{\sin(\frac{1}{x})}{x-1}\sqrt{x}$ is continuous

Find all intervals on which $f(x)$ is continuous if

$$f(x) = \begin{cases} x & \text{if } x \leq 3 \\ \frac{\sqrt{x^2-9}}{\sqrt{2x-6}} & \text{if } x > 3 \end{cases}$$