Lesson 5

Read 2.3 and 2.6

More limit calculations

The $\lim_{x \to *} f(x)$

1. If $* = x_0$ first check if f is continuous at x_0, if it is, the answer is $f(x_0)$
2. If f is not continuous at x_0 or $* = \infty$
   - Can you apply limit laws ?
   - Is the limit of the form $\frac{k}{0}$, with $k \neq 0$?
     In this case you can use limit laws if you have a limit of the form $\frac{k}{0^+}$ or $\frac{k}{0^-}$; otherwise, if the denominator sign does not stay positive or negative , close to the limit x_0, consider the limits from the right and the left separately

If f is not continuous at x_0 or $* = \infty$ and you cannot apply the limit laws because either some limit does not exists (ex $\lim_{x \to \infty} \sin x$) or you have an indeterminate form: $\infty - \infty$, $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$ you need to do something else (see next slide)

Limit calculations techniques

1. Algebra
2. Rational functions.
3. Rationalize
4. Squeeze theorem
5. De l 'Hospital (sec 4.4)

Suppose $\lim_{x\to 1} f(x) = 4$ and $\lim_{x\to 1} g(x) = -1$ Calculate

$\lim_{x\to 1}(f(x) + g(x))$

$\lim_{x\to 1} \sqrt{f(x)}$

Calculate
$\lim_{x \to 1} \frac{x-2}{x^2-4}$

$\lim_{x \to 2} \frac{x-2}{x^2-4}$

$\lim_{x \to 2} \frac{x-1}{(x-2)^2}$

$\lim_{x\to 2} \frac{x-1}{x-2}$

$\lim_{x\to \infty} \frac{x-1}{x-2}$

Question

Are $f(x) = \frac{x-2}{x^2-4}$ and $g(x) = \frac{1}{x+1}$ the same function ?

Can we interpret the limits that we have calculated in the previous pages graphically ?

Horizontal asymptote

The line $y = L$ is called an horizontal asymptote for $y = f(x)$ if either $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$

Vertical asymptote

The line $x = a$ is called a vertical asymptote for $y = f(x)$ if either $\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$

Rational functions

A rational function is a function of the form $f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots b_1 x + b_0}$, when we compute limits we are interested in looking at the highest power of x on top and the highest power of x on the bottom , so we can think of a rational function as $f(x) = \frac{a_n x^n + \cdots}{b_m x^m + \cdots}$

$\lim_{x \to \infty} \frac{a_n x^n + \cdots}{b_m x^m + \cdots}$

- If $n < m$ the limit is 0
- If $n > m$ the limit $+\infty$ if a_n and b_m have the same sign, $-\infty$ if a_n and b_m have opposite signs
- If $n = m$ the limit is $\frac{a_n}{b_n}$

Compute the following limit $\lim_{x \to \infty} \frac{x^2 - 4 + x}{x - 2}$

Compute the following limit $\lim_{x\to\infty} \frac{x^2-4+x}{x^2-2}$

Compute the following limit $\lim_{x\to\infty} \frac{x^2-4+x}{x^3-2}$

Rationalizing

Rationalizing often helps when calculating limits with terms of the form $a \pm \sqrt{x + a^2}$

Calculate $\lim_{x\to 0} \frac{1}{x\sqrt{x+1}} - \frac{1}{x}$

Squeeze theorem

If $g(x) \leq f(x) \leq h(x)$ for all x close to x_0 and $\lim_{x \to x_0} g(x) = \lim_{x \to x_0} h(x) = L$ then $\lim_{x \to x_0} f(x) = L$

Example

Calculate $\lim_{x \to 0} x^2 \sin(\frac{1}{x})$

Example

Calculate $\lim_{x \to 0} x \sin(\frac{1}{x})$