Lesson 4

Read 2.3 and 2.6

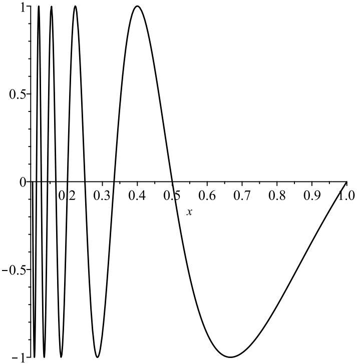
Limit calculations

Limits computation

- Look at graph.
- Table of values . Not so good.
- Continuity.

Example

Guess the value of $\lim_{x\to 0} \sin(\frac{\pi}{x})$ by making a table of values for $\sin(\frac{\pi}{x})$ with $x = 1, 0.1, 0.01, -1, -0.1, -0.01$


1
0.5
0
−0.5
−1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
x

Is it true that if f is defined at some point x_0 then $\lim_{x\to x_0} f(x) = f(x_0)$?

Continuity (more in 2.5)

A function f is $\boxed{\text{continuous}}$ at a point x_0 if:

- f is defined at x_0 and
- $\lim_{x \to x_0} f(x) = f(x_0)$
- Intuitive graphical explanation is that f is continuous at x_0 if f is defined at x_0 and the graph of f has no holes or gaps at x_0

A function is continuous on a set S if it is continuous for every x_0 in S.

A function is continuous if it is continuous on its domain.

Elementary functions

All the functions below are elementary functions

- Constants: $f(x) = c$
- Powers : $f(x) = x^n$ (n a whole number, positive ore negative)
- Roots: $f(x) = \sqrt[m]{x}$
- Exponentials: a^x
- Logs: $f(x) = \ln x$
- Trigonometric functions: $\sin x, \cos x \cdots$

All elementary functions are continuous.
Any function created by adding,subtracting, multiplying, dividing elementary functions is continuous.

Any function created by composing elementary functions is continuous.

So which functions we may see in this class may fail to be continuous ?

Functions defined by cases (multipart functions)

Note

$\frac{1}{x}$ is continuous

Back to limits

If f is a continuous function $\lim_{x \to x_0} f(x) = f(x_0)$

Example

$\lim_{x \to 3} \frac{x-1}{x-2} =$

Problems

- We want to compute $\lim_{x \to x_0} f(x)$ and f is not defined at x_0.
- We want to compute $\lim_{x \to x_0} f(x)$, f is defined at x_0, but not continuous at x_0.
- We want to compute $\lim_{x \to \infty} f(x)$ or $\lim_{x \to -\infty} f(x)$

$\lim_{x\to\infty} e^x =$ and $\lim_{x\to-\infty} e^x =$

$\lim_{x\to 2} \frac{x-1}{x-2} =$ and $\lim_{x\to\infty} e^x + arctanx =$

Limit laws from sec 2.3

Suppose $\lim_{x\to x_0} f(x)$ and $\lim_{x\to x_0} g(x)$ both exists and are finite, say $\lim_{x\to x_0} f(x) = a$ and $\lim_{x\to x_0} g(x) = b$ then

- $\lim_{x\to x_0}(f(x) + g(x)) = \lim_{x\to x_0} f(x) + \lim_{x\to x_0} g(x) = a + b$.
- $\lim_{x\to x_0}(f(x) - g(x)) = \lim_{x\to x_0} f(x) - \lim_{x\to x_0} g(x) = a - b$.
- $\lim_{x\to x_0}(f(x) \cdot g(x)) = \lim_{x\to x_0} f(x) \cdot \lim_{x\to x_0} g(x) = a \cdot b$.
- $\lim_{x\to x_0}(cf(x)) = c \lim_{x\to x_0} f(x) = ca$.
- If $b \neq 0$ $\lim_{x\to x_0} \frac{f(x)}{g(x)} = \frac{\lim_{x\to x_0} f(x)}{\lim_{x\to x_0} g(x)} = \frac{a}{b}$.

Note: the same laws hold if we replace x_0 with ∞ or $-\infty$

Example

$\lim_{x \to 1} \arctan x + \frac{1}{x} =$

Example

$\lim_{x \to \infty} \arctan x + \frac{1}{x} =$

Limit laws involving ∞

- $k + (+\infty) = (+\infty), k + (-\infty) = (-\infty)$, $(+\infty) + (+\infty) = (+\infty)$, $(-\infty) + (-\infty) = (-\infty)$
- $\boxed{(+\infty) + (-\infty) = ?}$
- $k \cdot (+\infty) = (+\infty)$ when $k > 0$,$k \cdot (+\infty) = (-\infty)$ when $k < 0$, $(+\infty) \cdot (+\infty) = (+\infty)$, $(+\infty) \cdot (-\infty) = (-\infty)$, $(-\infty) \cdot (-\infty) = (+\infty)$
- $\boxed{0 \cdot (\infty) = ?}$
- $\frac{k}{\infty} = 0$, k any number.
- $\boxed{\dfrac{\infty}{\infty} = ?}$
- $\boxed{\dfrac{0}{0} = ?}$

Example

$\lim_{x \to \infty} e^x + \arctan x =$

Limit laws involving 0 at the denominator

- $\frac{c}{0^+} = +\infty$ if $c > 0$
- $\frac{c}{0^+} = -\infty$ if $c < 0$
- $\frac{c}{0^-} = -\infty$ if $c > 0$
- $\frac{c}{0^-} = +\infty$ if $c < 0$
- $\boxed{\frac{0}{0} = ?}$

Example

Compute $\lim_{x \to 2} \frac{x-1}{x-2}$