Lesson 21

Read 4.4 and 4.5

Limits of functions of the form $f(x)^{g(x)}$

Function graphing

How to compute $\lim f(x)^{g(x)}$

1. Write $f(x)^{g(x)} = e^{\ln(f(x))\cdot g(x)}$
2. Calculate $\lim \ln(f(x)) \cdot g(x)$
3. Compute original limit (see table in next page)

$\lim \ln(f(x)) \cdot g(x)$	$\lim e^{\ln(f(x)) \cdot g(x)}$
DNE	DNE
$+\infty$	$+\infty$
$-\infty$	0
L	e^L

Calculate $\lim_{x \to \infty} x^x$

Function graphing

Horizontal asymptotes

The line $y = c$ is an horizontal asymptote for $y = f(x)$ is $\lim_{x \to +\infty} f(x) = c$ or $\lim_{x \to -\infty} f(x) = c$

Vertical asymptotes

The line $x = a$ is an horizontal asymptote for $y = f(x)$ is $\lim_{x \to a^+} f(x) = \infty$ or $\lim_{x \to a^-} f(x) = \infty$

Inflection points

An inflection point for $y = f(x)$ is a point $(c, f(c))$ on the curve $y = f(x)$ where the curve is continuous and changes concavity. If $f''(c) = 0$ and $f''(x) < 0$ for $x < c$ and $f''(x) > 0$ for $x > c$ or viceversa ($f''(x) > 0$ for $x < c$ and $f''(x) < 0$ for $x > c$) then $(c, f(c))$ is an inflection point for $y = f(x)$.

To graph $y = f(x)$

1. Determine the domain of f
2. x and y intercepts.
3. Compute horizontal and vertical asymptotes.
4. Compute $f'(x)$ and determine the intervals where f is increasing and decreasing.
5. Find the local minima and maxima for f.
6. Compute $f''(x)$ and determine the inflection points and the intervals where f is concave up and down.

Draw the graph of $f(x) = xe^{-x^2}$
Domain : for which values of x can we compute $f(x)$?

Horizontal asymptotes :
Compute $\lim_{x \to -\infty} f(x)$ if the domain of f allows it

Compute $\lim_{x \to \infty} f(x)$ if the domain of f allows it

Vertical asymptotes :
if f is not defined at $x = a$ but it is defined to the left of a
compute $\lim_{x \to a^-} f(x)$

if f is not defined at $x = a$ but it is defined to the right of a
compute $\lim_{x \to a^+} f(x)$

Compute $f'(x)$, intervals of increase, decrease, local min and max

Compute $f''(x)$, inflection points and intervals where f is concave up and down .

Find x and y intercepts

Draw graph