Lesson 20

Read 4.4

De l Hospital rule

When

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

results in an indeterminate form $\frac{0}{0}$ or $\frac{\infty}{\infty}$
if $g^{'}(x)$ is different from 0 around a, except possibly at a you can calculate

$$\lim_{x \to a} \frac{f^{'}(x)}{g'(x)}$$

and if this latter limit exists then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim \frac{f^{'}(x)}{g'(x)}$$

Explanation

Compute $\lim_{x \to \infty} \frac{x}{\ln x}$

Compute $\lim_{x\to\infty} \frac{e^x}{x}$

Compute $\lim_{x\to\infty} \frac{e^x}{x^2}$

o notation

We write $\ln(x) = o(x)$ (for $x \to \infty$) to mean that $\lim_{x\to\infty} \frac{\ln x}{x} = 0$.
Intuitively this says $\ln x$ is much smaller than x for "'big"' values of x

In general we write $f(x) = o(g(x))$ (for $x \to \infty$) to mean that $\lim_{f(x)\to\infty} \frac{f(x)}{g(x)} = 0$.
Intuitively this says $f(x)$ is much smaller than $g(x)$ for "'big"' values of x

NOTE

every function in the list below is o() of the function(s) to its right:

$$\sin x \text{and} \cos x, \ln x, (\ln x)^n (n > 1), \sqrt{x}, x, x^n (n > 1), e^x$$

Compute $\lim_{x\to\infty} \frac{(\ln x)^{10}}{x^5}$

Compute $\lim_{x\to\infty} \frac{(x+\sin x)\ln x}{xe^x}$

Compute $\lim_{x \to \infty} x - \ln x$

Compute $\lim_{x \to 0^+} \frac{x}{\ln x}$

Compute $\lim_{x \to \frac{\pi}{2}} \frac{x - \frac{\pi}{2}}{\cos x}$

What is the linearization of $\cos x$ at $x = \frac{\pi}{2}$?

Compute $\lim_{x \to 0} \frac{7 \sin x - 7x}{x^3}$

What is the linearization of $\sin x$ at $x = 0$?

Compute $\lim_{x \to 0} \frac{7 \sin x - 7x}{x^3}$ using o()