Lesson 2

Read 2.1

Tangent lines

Average and instantaneous velocity

Average and instantaneous rate of change

Common misconceptions on tangent lines

A curve and its tangent at some point P intersect only at P. NOT ALWAYS TRUE.

The tangent to a curve at P is perpendicular to OP.NOT ALWAYS TRUE

Tangent to a general curve $y = f(x)$

Given two points on the curve $P = (x_1, f(x_1))$ and $Q = (x_2, f(x_2))$ the slope of the line through P and Q is $\frac{f(x_2)-f(x_1)}{x_2-x_1}$

What happens when we move x_2 closer and closer to x_1? Maple animation.

When we move x_2 closer and closer to x_1 $\frac{f(x_2)-f(x_1)}{x_2-x_1}$ gets closer and closer to the slope of the tangent line to the curve $y = f(x)$ at P .

Average velocity

If an object moves on a straight line and its position at time t (with respect to a given origin O) is given by $s(t)$ then its average velocity in the time interval $[t_1, t_2]$ is given by (distance traveled)/time=

$$v = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Instantaneaous velocity

How can we calculate the instantaneous velocity at time t ?

Geometric interpretation

Consider the curve $y = s(t)$,

- The average velocity in the time interval $[t_1, \ t_2]$ is $v = \frac{s(t_2)-s(t_1)}{t_2-t_1}$ and can be interpreted as the slope of the line that goes through the points $P = (t_1, s(t_1))$ and $Q = (t_2, s(t_2))$
- The instantaneous velocity at t_1 is $v(t_1) = \lim_{t_1 \to t_2} \frac{s(t_2)-s(t_1)}{t_2-t_1}$ and it can be interpreted as the slope of the line tangent to the curve $y = s(t)$ at the point $P = (t_1, s(t_1))$

Rate of change of a function

The average rate of change of any function $y = f(t)$ over the interval $[t_1,\ t_2]$ is given by $\frac{f(t_2)-f(t_1)}{t_2-t_1}$

The instantaneous rate of change of any function $y = f(t)$ at t_1 is given by $\lim_{t_2 \to t_1} \frac{f(t_2)-f(t_1)}{t_2-t_1}$

Geometrically the average rate of change above is the slope of the line going through the points $P = (t_1, f(t_1))$ and $Q = (t_2, f(t_2))$.

Geometrically the instantaneous rate of change above is the slope of the line tangent to the curve $y = f(t)$ at the point $P = (t_1, f(t_1))$

Assignment Previewer

hw02S2.1 (9966474)

Previewer Tools

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Show: All, None ☑ Assignment Score ☐ Key ☐ Solution ☐ Help/Hints ☐ Mark ☐ Answer Format Tips

Current Score: 0/68 **Due:** Mon Jan 9 2017 11:00 PM PST

Question	1	2	3	4	5	6	7	8	Total
Points	0/10	0/12	0/11	0/10	0/14	0/4	0/4	0/3	0/68

1. **0/10 points** SCalcET7 2.1.007. [1608988] -

The table shows the position of a cyclist.

t (seconds)	0	1	2	3	4	5
s (meters)	0	1.3	5.5	10.6	17.9	25.6

(a) Find the average velocity for each time period.

(i) [1, 3]
____ m/s

(ii) [2, 3]
____ m/s

(iii) [3, 5]
____ m/s

(iv) [3, 4]
____ m/s

(b) Estimate the instantaneous velocity when $t = 3$.
____ m/s

2. **0/12 points** SCalcET7 2.1.001.MI. [3235104] -

A tank holds 5000 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table show the volume V of water remaining in the tank (in gallons) after t minutes.

t (min)	5	10	15	20	25	30
V (gal)	3475	2225	1275	545	125	0

(a) If P is the point (15, 1275) on the graph of V, find the slopes of the secant lines PQ when Q is the point on the graph with the following values. (Round your answers to one decimal place.)

Q	slope
(5, 3475)	
(10, 2225)	
(20, 545)	
(25, 125)	