Lesson 19

Read 4.1 , 4.3

Global min and max on $[a, b]$

Local min and max: how to classify critical numbers

Find the global min and the global max for $f(x) = \frac{x}{x^2+1}$ on $[0, 2]$

Find the global min and the global max for $f(x) = \frac{|x|}{x^2+1}$ on $[-2, 2]$

Critical numbers

The local max and min of a functions must be critical numbers, but the vice-versa is not true

A critical number can be :

- A local max. Ex $x = 0$ for $f(x) = -x^2$.
- A local min . Ex $x = 0$ for $f(x) = x^2$.
- Neither . Ex $x = 0$ for $f(x) = x^3$.

How to classify critical numbers

The first derivative test

If c is a critical number for a continuous function f

- if f' is positive in some interval (a, c) and negative on some interval (c, b) then c is a local max.
- if f' is negative in some interval (a, c) and positive on some interval (c, b) then c is a local min.
- if f' does not change sign at c c is neither max nor min.

Warning : f must be continuous

Example

Consider

$$f = \begin{cases} x & \text{if } x \leq 1 \\ -x + 5 & \text{if } x > 1 \end{cases}$$

and look at $x = 1$

The second derivative test

- if $f''(c)$ is positive then c is a local min.
- if $f''(c)$ is negative then c is a local max.
- if $f''(c)$ is zero the test cannot be used.

How to find all local minima and maxima for f

- Compute f'.
- List all critical numbers.
- Use either the first or second derivative test to classify the critical numbers..

Find and classify all critical numbers of $f(x) = \frac{x^3}{3} - \frac{5}{2}x^2 + 6x$ on $(-\infty, \infty)$

Find and classify all critical numbers of $f(x) = \cos x + \cos^2 x$ on $[0, 2\pi]$.

Find the global min and global max of $f(x) = \cos x + \cos^2 x$ on $[0, 2\pi]$.