

Lesson 18

Read 4.1

Global min and max on $[a, b]$

Terminology

Given a function $f(x)$

- ▶ an open interval is an interval of the form (a, b) (a and b are allowed to be ∞)
- ▶ a closed interval is an interval of the form $[a, b]$
- ▶ the domain of f consists of all allowed values of x .
- ▶ a local maximum is a number c (x value) that is in the domain of f , but, if the domain is a closed interval $[a, b]$, it is not a nor b , such that there is an open interval I containing c and contained in the domain of f s.t. for all x in I we have $f(c) \geq f(x)$.
- ▶ a local maximum value is a value $y = f(c)$ when c is a local maximum

- ▶ a local minimum is a number c (x value) that is in the domain of f , but , if the domain is a closed interval $[a, b]$, it is not a nor b , such that there is an open interval I containing c and contained in the domain of f s.t. for all x in I we have $f(c) \leq f(x)$.
- ▶ a local minimum value is a value $y = f(c)$ when c is a local minimum.
- ▶ A critical number is a number c in the domain of f , s.t. either $f'(c) = 0$ or $f'(c)$ does not exist

- ▶ a global or absolute maximum is a number c (x value) that is in the domain of f such that s.t. for all x in I we have $f(c) \geq f(x)$.
- ▶ the global or absolute maximum value is the value $y = f(c)$ when c is a global maximum.
- ▶ a global or absolute minimum is a number c (x value) that is in the domain of f such that s.t. for all x in I we have $f(c) \leq f(x)$.
- ▶ the global or absolute minimum value is the value $y = f(c)$ when c is a global minimum.

local max =
global max =
local min =
global min =
critical points =

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local max =
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local max =
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global min =
critical points =

local max value =
global max value =
local min value =
global min value =

Important facts about local max and min

- ▶ All local min and max of a function are critical points for the function
- ▶ A function may have critical points that are neither local min nor max

Important facts about global max and min

- ▶ All global max and min of a function are either endpoints of the domain of the function or local max and local min respectively the function
- ▶ A continuous function defined on a closed and bounded interval $[a, b]$ always has a global max and a global min.
- ▶ If f is a continuous function defined on $[a, b]$ then the global max or global min are either a or b or critical number for f .

Global max and min calculations

To find the global max and min of a continuous function defined on a closed and bounded interval $[a, b]$

- ▶ Find all critical number of f
- ▶ Calculate all values of f at a , b and all critical points.
- ▶ The x value that gives the biggest f value in the computation above is the global max, the x value that gives the smallest f value in the computation below is the global min.

Find all critical points for $f(x) = \frac{x^3}{3} - \frac{5}{2}x^2 + 6x$

- ▶ Calculate $f'(x)$

- ▶ Solve for $f'(x) = 0$

- ▶ Find all values of x for which $f'(x)$ is not defined.

Find the global min and the global max for $f(x) = \frac{x^3}{3} - \frac{5}{2}x^2 + 6x$ on $[0, 10]$

Find the global min and the global max for $f(x) = \frac{x^3}{3} - \frac{5}{2}x^2 + 6x$ on $[0, 1]$