Lesson 16

Read 3.10

Tangent line approximation

Recall that given y=f(x) the equation of the tangent line at $P = (a, f(a))$ is given by

$$y = f(a) + f^{'}(a)(x - a)$$

Consider the function $g(x) = f(a) + f^{'}(a)(x - a)$, g is called the $\boxed{\text{linearization}}$ or $\boxed{\text{linear approximation}}$ or $\boxed{\text{tangent line approximation}}$ of f at a

g approximates f in a "'small"' interval around a

Find the linearization of $f(x) = \sin x$ at 0.

- If f is concave up at a its linearization at a is an underestimate.

- If f is concave down at a its linearization at a is an overestimate.

Estimate the cubic root of 8.1. Do you think you gave an overestimate or an underestimate ?

Argue that the equation $x^3 + 2x = 0.1$ has a solution. Estimate the solution

differentials

If g is the linearization of f at a we write :

$$
\begin{aligned}
g(x) &= f(a) + f^{'}(a)(x-a) \text{ and } & f(x) &\approx g(x) \\
g(x_2) &= f(a) + f^{'}(a)(x_2-a) \text{ and } & f(x_2) &\approx g(x_2) \\
g(x_1) &= f(a) + f^{'}(a)(x_1-a) \text{ and } & f(x_1) &\approx g(x_1) \\
g(x_2) - g(x_1) &= f^{'}(a)(x_2-x_1) \text{ and } & f(x_2) - f(x_1) &\approx g(x_2) - g(x_1) \\
g(x_2) - g(x_1) &= f^{'}(a)(x_2-x_1) \text{ and } & f(x_2) - f(x_1) &\approx f^{'}(a)(x_2-x_1) \\
g(x_2) - g(x_1) &= f^{'}(a)(x_2-x_1) \text{ and } & dy &\approx f^{'}(a)dx
\end{aligned}
$$

Suppose you are trying to measure the area of a circle by measuring the radius r and using the formula $A(r) = \pi r^2$. Your instrument measuring the radius has an error of ± 0.05 cm. What is the error you make in calculating the area, if you measure $r = 10$?

A right circular cone of height h and base radius r has a total surface area consisting of its base area πr^2 plus its side area $\pi r\sqrt{r^2+h^2}$. Suppose you start out with a cone of height 8 cm and base radius 6 cm , and you want to change the dimension in such a way that the total surface area remains the same. If you change the height to 8.04 cm, what is your new value for the base radius ? Use implicit diferentiation and the tangent line approximation.

A particle is traveling along a curve with parametric equations

$$\begin{aligned} x &= x(t) \\ y &= y(t) \end{aligned}$$

The implicit equations of the curve is $y^2 = x^3 + 3x$. At time $t = 0$ the particle is located at (1,-2) and its vertical velocity $\frac{dy}{dt} = 2$ units/sec. Use the tangent line approximation to estimate the location of the particle at time $t = 0.1$