Lesson 15

Read 3.5, 3.6

Logarithmic differentiation

Derivatives of inverse functions

formulas to know

- $(\ln x)' = \frac{1}{x}$
- $(\arctan x)' = \frac{1}{1+x^2}$
- $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
- $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$

Derivative of the inverse of a function

In order to calculate $(f^{-1})'$

- Consider $y = f^{-1}(x)$ so then $f(y) = x$; write instead $y = y(x)$, and $f(y(x)) = x$. (*) .
- Take the derivative with respect to x of both sides of (*).
- You get $\frac{df}{dy} \frac{dy}{dx} = 1$.
- You get $\frac{dy}{dx}(x) = \frac{1}{\frac{df}{dy}(y)}$. Simplify and get a formula in x

Example. Derive the formula $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

Example. Derive the formula $(\ln x)' = \frac{1}{x}$

Calculate the derivative of $\ln(|x|)$

Calculate the derivative of $\ln(3x + \ln(3x + \ln(3x)))$

Logarithmic differentiation

This method is used to find derivatives of functions of the form $g(x)^{h(x)}$

- Write $y = g(x)^{h(x)}$ and take ln of both sides.
- You get $\ln(y) = h(x)\ln(g(x))$. Now differentiate both sides with respect to x.
- You get $\frac{1}{y}y^{'} = h^{'}\ln g + h\frac{g^{'}}{g}$. Solve for $y^{'}$.
- You get $y^{'} = y(h^{'}\ln g^{'} + h\frac{g^{'}}{g})$. Replace y with $g(x)^{h(x)}$.
- Your final answer is $y^{'}(x) = g(x)^{h(x)}(h^{'}(x)\ln(g(x)) + h(x)\frac{g^{'}(x)}{g(x)})$.

Alternative method to calculate derivatives of functions of the form $g(x)^{h(x)}$

Write $f(x) = g(x)^{h(x)}$ so $f(x) = e^{\ln g(x) h(x)}$ and use the chain rule.

Calculate the derivative of $f(x) = x^x$

Calculate the derivative of $f(x) = (x+1)^{\cos x \sin x}$

Write the equations of the tangent lines to the ellipse
$C = \frac{x^2}{25} + \frac{y^2}{9} = 1$ at $P = (5\frac{\sqrt{3}}{2}, \frac{3}{2})$

Write the equations of the tangent lines to the curve

$$x = 5\sin(\pi t)$$
$$y = 3\cos(\pi t)$$

at $P = (5\frac{\sqrt{3}}{2}, \frac{3}{2})$