# Lesson 13

Read 3.5

Implicit differentiation

Recall $\frac{d}{dx}(\sin x)^2 = 2\sin x\cos x$

In general

$\frac{d}{dx}(y(x))^2 =$

# Curve defined implicitly

A curve is defined implicitly if it is defined by a formula of the form $F(x, y) = G(x, y)$.
An example of a curve defined implicitly is the circle of radius 1 defined by the formula $x^2 + y^2 = 1$.
We can think that the equation $F(x, y) = G(x, y)$ defines a function $y = y(x)$ even if we are not able to solve explicitly for $y$.
To find $\frac{dy}{dx}$ pretend we can solve for $y$ and write $F(x, y(x)) = G(x, y(x))$, then take the derivative with respect to $x$ of both sides and solve for $\frac{dy}{dx}$.
The equation of the tangent line to the curve defined implicitly by $F(x, y) = G(x, y)$ at $P = (x_0, y_0)$ on the curve is $y = y_0 + \frac{dy}{dx}(x_0)(x - x_0)$.

Find the equation of the tangent line to the curve $x^3 + y^3 = 6xy$ at $P = (3, 3)$

Find $\frac{dy}{dx}$ by implicit differentiation if $\sqrt{xy} = 1 + x^2 y$

Find the coordinates of all points $P$ on the curve $x^2 + 2xy + y^3 = 0$ such that the tangent line to the curve at $P$ is horizontal.

Given a curve $C$ of equation $F(x, y) = G(x, y)$, to find all points on $C$ with horizontal tangent first use implicit differentiation to calculate the derivative, say you found the formula $\frac{dy}{dx} = h(x, y)$, then solve the system,

$$\begin{aligned} F(x, y) &= G(x, y) \\ h(x, y) &= 0 \end{aligned}$$

If $h(x, y) = \frac{A(x,y)}{B(x,y)}$, to find all points on $C$ with vertical tangent line solve the system

$$\begin{aligned} F(x, y) &= G(x, y) \\ B(x, y) &= 0 \end{aligned}$$

and check that for every solution $(x_1, y_1)$ we have $A(x_1, y_1) \neq 0$ (if it is 0 we are not sure what is happening to the tangent)

# The tangent line problem for $P$ is not on the curve

To find the equation to the tangent line to the curve $F(x, y) = G(x, y)$ through a point $P = (x_0, y_0)$ NOT on the curve:

- Call $Q = (x, y)$ the unknown point of tangency on the curve.
- Write the equation of the slope of the tangent $m$ in two different ways , set them equal $m = \frac{dy}{dx} = \frac{y-y_0}{x-x_0}$, and solve the system

$$\frac{dy}{dx} = \frac{y - y_0}{x - x_0}$$
$$F(x, y) = G(x, y)$$

- If $(x_1, y_1)$ is a solution you found, then a tangent line is $y = y_1 + \frac{dy}{dx}(x_1)(x - x_1)$

Find all tangents to the ellipse $x^2 + y^2 = 1$ through $P = (4, \frac{1}{4})$

Find $y''$ by implicit differentiation if $9x^2 + y^2 = 9$