Lesson 13

Read 10.2

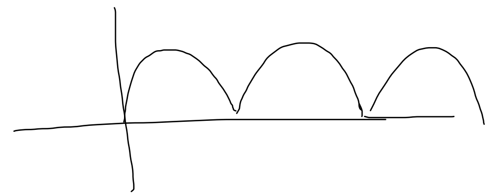
parametric curves , slopes and tangent lines

Find all the ~~points Q~~ times t such that the tangent line to the curve C of the previous slide is horizontal. at t

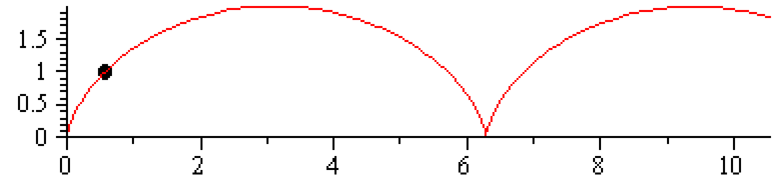
$x = t - \sin t$

$y = 1 - \cos t$

$t > 0$



What is the equation of the tangent line to C at P ?



The tangent to the curve described by the parametric equations

$$
\begin{aligned}
x &= x(t) \\
y &= y(t) \quad a \leq t \leq b
\end{aligned}
$$

is horizontal for all t s.t $\frac{dy}{dt}(t) = 0$ and $\frac{dx}{dt}(t) \neq 0$
is vertical for all t s.t $\frac{dx}{dt}(t) = 0$ and $\frac{dy}{dt}(t) \neq 0$
when they are both 0 we are not sure , we can try to use limits to find out as in the previous example

The tangent line problem for parametric equations when P is not on the curve

To find the equation to the tangent line to the curve

$$x = x(t)$$
$$y = y(t) \quad a \leq t \leq b$$

through a point $P = (x_0 = x(t_0), y_0 = y(t_0))$ NOT on the curve:

- Call $Q = (x(t), y(t))$ the unknown point of tangency on the curve.
- Write the equation of the slope of the tangent m in two different ways , set them equal, and solve for t

$$m = \frac{\frac{dy}{dt}(t)}{\frac{dx}{dt}(t)} = \frac{y(t) - y(t_0)}{x(t) - x(t_0)}$$

- If $t = t_1$ is a solution you found, then a tangent line is $y = y(t_1) + \frac{\frac{dy}{dt}(t_1)}{\frac{dx}{dt}(t_1)}(x - x(t_1))$

Find the equation of the tangent line to the curve

$$
\begin{aligned}
x &= 3t^2 + 2 \\
y &= 4t^3 + 2 \quad t > 0
\end{aligned}
$$

going through $Q = (6, 2)$.

2. (16 Points) An object is moving in the plane with parametric equations

$$x(t) = \sin(t) \qquad y(t) = 3\cos(t) \qquad \text{for time} \quad 0 \leq t \leq 2\pi$$

The time is measured in seconds and the coordinate axes have units of feet.

(a) Find the equation of the tangent line when $t = \pi/3$ seconds.

(b) Let $s(t)$ be the speed of the object at time t. Calculate $s(\pi/3)$; make sure to include units.

(c) Is the speed of the object increasing or decreasing at time $\pi/3$? You must justify your answer.

(d) Suppose the object leaves the path at time $t = \pi/3$ and continues travelling along the tangent line (recall part (a)) with constant speed $s(\pi/3)$ (recall part (b)). Where is the object located 2 seconds later?