Lesson 12

Read 10.2

Parametric curves calculus. Tangent lines only

Recall: parametric equations for a curve are a pair of equations of the form:

$$\begin{aligned} x &= x(t) \\ y &= y(t) \quad a \leq t \leq b \end{aligned}$$

Recall that the tangent line at $P = (x_1, y_1 = f(x_1))$ to the curve $y = f(x)$ is given by

$$y = y_1 + \frac{df}{dx}(x_1)(x - x_1)$$

Tangent lines to curves described by parametric equations

Pretend the parametric curve in the previous slide is also described by the formula $y = f(x)$, then we can write

$$y(t) = f(x(t))$$

Tangent line formula

The slope of the tangent to the curve described by the parametric equations

$$\begin{aligned} x &= x(t) \\ y &= y(t) \quad a \leq t \leq b \end{aligned}$$

at $P = (x_1 = x(t_1), y_1 = y(t_1))$ is

$$m = \frac{\frac{dy}{dt}(t_1)}{\frac{dx}{dt}(t_1)}$$

Example

Consider the curve C

$$x = t - \sin t$$
$$y = 1 - cost \quad t > 0$$

Find the equation of the line tangent to C at the point $P(\frac{\pi}{2} - 1, 1)$

Find all the points Q such that the tangent line to the curve C of the previous slide is horizontal.

The tangent to the curve described by the parametric equations

$$x = x(t)$$
$$y = y(t) \quad a \leq t \leq b$$

is horizontal for all t s.t $\frac{dy}{dt}(t) = 0$ and $\frac{dx}{dt}(t) \neq 0$
is vertical for all t s.t $\frac{dx}{dt}(t) = 0$ and $\frac{dy}{dt}(t) \neq 0$
when they are both 0 we are not sure , we can try to use limits to find out as in the previous example

The tangent line problem for parametric equations when P is not on the curve

To find the equation to the tangent line to the curve

$$x = x(t)$$
$$y = y(t) \quad a \leq t \leq b$$

through a point $P = (x_0 = x(t_0), y_0 = y(t_0))$ NOT on the curve:

- Call $Q = (x(t), y(t))$ the unknown point of tangency on the curve.
- Write the equation of the slope of the tangent m in two different ways , set them equal, and solve for t

$$m = \frac{\frac{dy}{dt}(t)}{\frac{dx}{dt}(t)} = \frac{y(t) - y(t_0)}{x(t) - x(t_0)}$$

- If $x = t_1$ is a solution you found, then a tangent line is $y = y(t_1) + \frac{\frac{dy}{dt}(t_1)}{\frac{dx}{dt}(t_1)}(x - x(t_1))$

Find the equation of the tangent line to the curve

$$x = 3t^2 + 2$$
$$y = 4t^3 + 2 \quad t > 0$$

going through $P = (6, 2)$.