Lesson 1

Read 10.1

Parametric equations

- web page : https://sites.math.washington.edu/ ep2/classes/124/124.html
- Math Department page for 124 (math 124 materials website): https://sites.math.washington.edu/ m124/
- email : ep2@uw.edu
- announcements
- syllabus
- Exam dates
- WebAssign
- Lectures before /after
- Worksheet

Precalculus review

- Lines
- Circles
- Tangent to a circle
- Parametric equations of motion (uniform linear motion)
- Uniform circular motion

Handouts on the web

Parametric equations

A pair of equations

$$x = f(t)$$
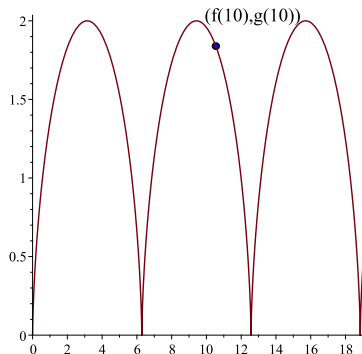$$y = g(t) \quad a \leq t \leq b \text{ (sometimes we write this as } t \in [a, b])$$

Describes a 2D curve consisting of all points of the form
$P = (f(t), g(t))$ for $t \in [a, b]$

Example

$$x = t - \sin t$$
$$y = 1 - \cos t \quad 0 \le t \le 20$$

Describes the curve below :

The same pair of equations

$$x = f(t)$$
$$y = g(t) \quad a \le t \le b$$

Also describes an object moving in 2D, that is gives information about the trajectory, starting position and velocity of the object.

Linear motion

Alice runs along a straight line L. She is at $P(1, 2)$ at time $t = 0$ and at $Q(-3, 5)$ at $t = 5$. Find Alice 's equation of motion.

Parametric equations of a circle

$$x = x_0 + r \cos t$$
$$x = y_0 + r \sin t \quad 0 \le t \le 2\pi$$

Parametric equation of a circle centered at $C = (x_0, y_0)$ and of radius r

More general parametric equations of a circle

$$x = x_0 + r\cos(\omega t + \alpha)$$
$$x = y_0 + r\sin(\omega t + \alpha) \quad 0 \le t \le T$$

Describe the motion of an object going around a circle centered at $C = (x_0, y_0)$ and of radius r, with angular velocity ω starting at position $P = (x_0 + r\cos(\alpha), x_0 + r\sin(\alpha))$

Uniform circular motion : motion of an object moving around a circle with constant velocity

- Angular velocity ω: speed at which angle θ swept by radius changes. $\boxed{\theta = \omega t}$
- Linear velocity v : velocity of the moving object. $\boxed{v = \omega r}$
- Period T: time it takes the object to compute a full revolution . $\boxed{T = \dfrac{2\pi}{\omega}}$

Example

Write the parametric equations of an object moving around the circle of radius 5, centered at the origin, with angular velocity $\omega = 2$ radians per second , starting at the point $P = (0, 5)$

Suppose that the motion of an object is described by the equations

$$x = 2 + \cos(2t + \pi)$$
$$y = -1 + \sin(2t + \pi) \quad 0 \le t \le \pi$$

Describe how the object moves.

Describe the curve

$$x = 2\cos(2t + \pi)$$
$$y = 3\sin(2t + \pi) \quad 0 \le t \le \pi$$