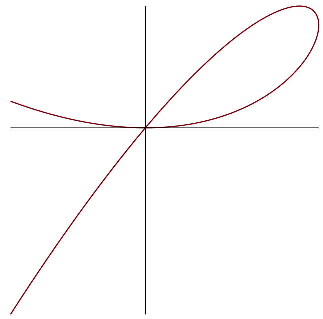
From Spring 15

5. (14 points) Consider the curve given by the implicit equation $x^3 - 4xy + y^2 = 0$.



(a) Find all points (a, b) on the curve where the tangent line is vertical.

(b) Check that the point $(3, 3)$ lies on the curve. Use a linear approximation to estimate the x-coordinate of a point on the curve with y-coordinate equal to 2.95. *Show all steps clearly.*

2. (12 total points) Compute the following limits. Your answers should be one of ∞, $-\infty$, DNE or a number. If you conclude that the limit does not exist, explain why not.

   (a) (4 points) $\displaystyle\lim_{x\to 1^+}\left(\frac{1}{x-1}-\frac{1}{(x-1)^2}\right)$

   (b) (4 points) $\displaystyle\lim_{x\to\infty}\sqrt{x+\sqrt{5x}}-\sqrt{x}$

   (c) (4 points) $\displaystyle\lim_{x\to 1}\frac{(x^2-4x+3)(x^2+x+1)}{x^2+x-2}$

7. (12 points) Consider the curve given by the following parametric equations:

$$x(t) = 9t - t^3 + 2\ ,\ \ y(t) = 3t^3 - t^4 + 1.$$

The curve passes through the point $(2, 1)$ for two different values of t.
Find the equations of the two tangent lines to the curve at the point $(2, 1)$.

8. (16 total points) Consider the function $f(x) = \dfrac{x^3}{(x+1)^2}$.

   (a) (2 points) Give the horizontal asymptotes of the graph of $y = f(x)$, if there are any.

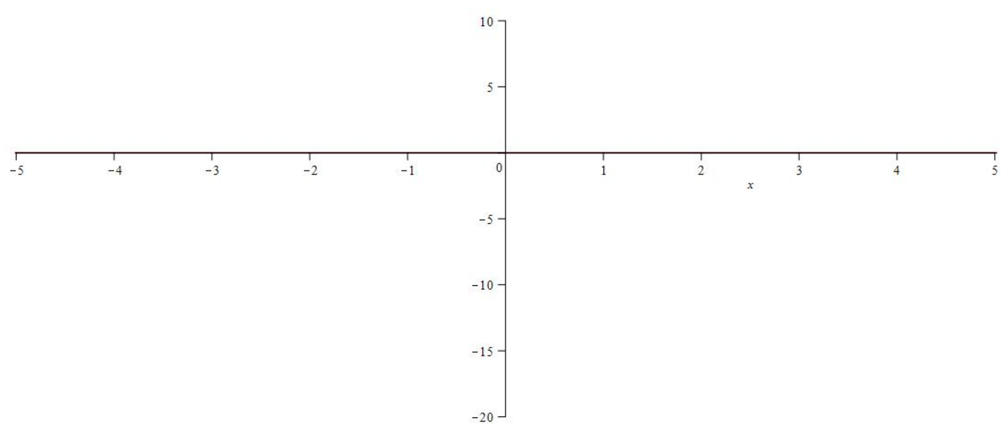
   (b) (2 points) Give the vertical asymptotes of the graph of $y = f(x)$, if there are any.

   (c) (4 points) Find the critical numbers for $f(x)$ and determine if each gives a local minimum, a local maximum, or neither.

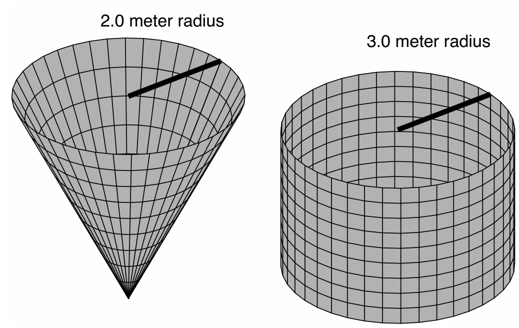
8. (continued) Recall the function $f(x) = \dfrac{x^3}{(x+1)^2}$

(d) (4 points) Does the graph of $y = f(x)$ have any inflection points? In which interval(s) is the graph concave up?

(e) (4 points) Sketch the graph of $y = f(x)$ on the axes provided below. Be sure to include asymptotes (if any) in your picture. Also, mark the coordinates of any local maximum, local minimum or inflection point. Make sure your picture matches the information you provided in parts (a)-(d).



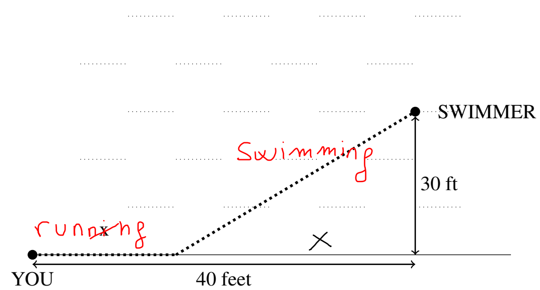
5. (14 points) Tank A has the shape of an inverted cone. It has height 8 meters and the radius at the top is 2 meters. It is full of water. Tank B has the shape of a cylinder with circular base of radius 3 meters. It is empty. The water is to be pumped from Tank A into Tank B.



The water level in Tank A is dropping at a rate of 15 centimeters/minute when the height of the water is 5 meters. How fast is the water rising in Tank B at that time?

Recall that the volume of a right circular cone is $\frac{1}{3}\pi r^2 h$.

5. (10 points) You are standing on the beach right by the water when you notice a swimmer in distress.He is 30 feet from shore, 40 feet from where you are standing. You can run twice as fast as you can swim. At what distance x from the place you are currently standing should you enter the water in order to minimize the time it takes you to reach the swimmer ?



1. (16 points)

   (a) Evaluate the following limits. Show your work. If you conclude that a limit does not exist, justify your answer.

      i. $\lim\limits_{x\to\infty}\left(x-\sqrt{x^2+4x}\right)$

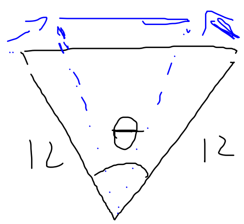
      ii. $\lim\limits_{x\to 0}\dfrac{e^x-\sin(x)-1}{x^4+7x^2}$

   (b) Compute the derivatives of the following functions:

      i. $y=\cos(\ln(\arcsin(x)))$

      ii. $g(x)=x^{\frac{1}{1+\ln(x)}}$

Mary is holding a fan having the shape of an isosceles triangle with two equal sides 12 in.long. She slowly starts to close the fan. How fast is the area of the fan decreasing when θ is $\frac{\pi}{4}$ (see picture below) and decreasing at a rate of $\frac{1}{3}$ rad /sec ?



You are making pizza. Initially your pizza is a circle of radius 4 and the slice shown in the picture below has an area of 5π in^2. Slowly you start stretching the dough, keeping it in a circular shape. When the radius of the pizza is 8 in and increasing at a rate of 0.1 in/sec , how fast is the area of the slice increasing ?