3. (14 points) A curve in the xy-plane is defined by the parametric equations

$$x(t) = 3t^2 + 1, \qquad y(t) = 2t^3 + 1$$

for all values of t.

(a) At what point on this curve does the tangent line have slope 3?

(b) Find the equations of all tangent lines to this curve that pass through the point $(5, 1)$. Give your answers in the form $y = mx + b$.

1 (12 points) Compute the derivatives of the following functions. Do not simplify your answers.

(a) (4 points) $g(x) = (2x+1)\sec\sqrt{3-5x}$

(b) (4 points) $h(x) = \sqrt{\sin\left(2x + \sin\sqrt{1+x^3}\right)}$

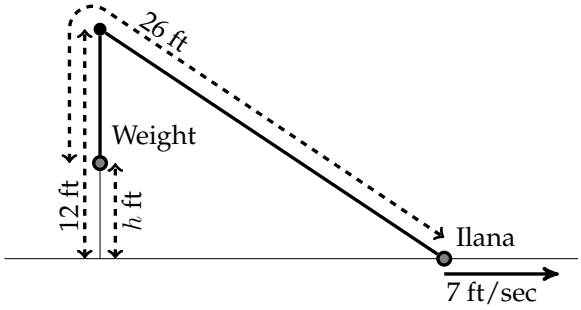
(c) (4 points) $y = \left(\tan^{-1} x\right)^{\ln x}$

6. **[15 points]** Ilana runs along the ground at a speed of 7 feet per second, while carrying one end of a 26-foot-long rope. The other end of the rope is tied to a weight through a pulley system. The top of the pulley is 12 feet off the ground, so the weight rises higher in the air as Ilana runs away from the pulley.

   When the weight is six feet off of the ground, how quickly is it rising into the air?

   In other words, let h be the height of the weight above the ground. Find $\frac{dh}{dt}$ when $h = 6$.

   (Ignore Ilana's height; you can pretend that she's zero feet tall, or that she's holding onto the rope by tying it to her shoe.)



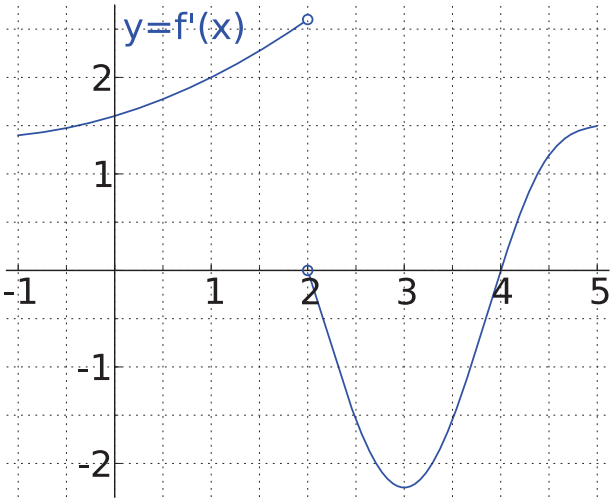
Problem 3 : Given the curve $C : x^2 + y^2 + xy = 7$

a) Find the tangent line to C at the point $P = (2, 1)$

b) Use the tangent line approximation to estimate the y value of a point $(2.01, y)$ on C

c) Does the value you calculated in b) underestimate or overestimate the exact value of y ? (Hint: is C concave up or down close to the point P?).

2. (20 points; 4pts each) TRUE/FALSE Below is the graph of the DERIVATIVE of a continuous function $f(x)$.



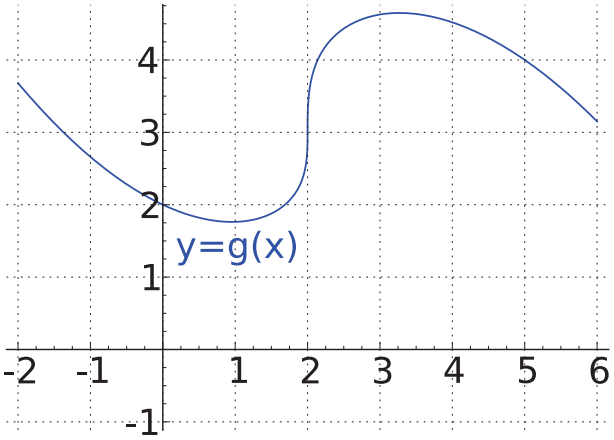
T=True, F=False.

a. The function f is increasing on the interval $(3, 4)$.

b. $f''(4) > 0$.

c. There is a horizontal tangent line to the graph of $y = f(x)$ at $x = 3$.

Below is the graph of a continuous function $g(x)$.



d. The graph of $y = f(g(x))$ has a horizontal tangent line at $x = 5$.

e. The graph of $y = f(g(x))$ has a horizontal tangent line at $x = 1$.