Here are a few problems from past midterms. I do not claim this review to be all inclusive.

The midterm will cover all material covered in class up to section 3.3

There will be no parametric equations problems.

1. Calculate the following limits. Your answer should be either a number, or $+\infty$, or $-\infty$ or DNE (does not exist). Make sure to justify all steps.

   (a) (5 points) $\lim_{x\to 1} \dfrac{x^2-1}{x^2-2x+1}$

   (b) (5 points) $\lim_{x\to 0} 1 + x^2 \cdot \sin\dfrac{1}{x} =$

   (c) (5 points) Find all values of the constant parameter A such that

$$\lim_{x\to+\infty} \frac{Ax^2-3}{7x^2-3x+2} = 2$$

d) $\lim_{x\to+\infty} \dfrac{\sqrt{14x^2-3}}{7x^2-3x+2}$

$$\lim_{x \to 1^-} \frac{1 - \sqrt{2-x}}{x-1}$$

$$\lim_{x \to 1^-} \frac{\cos x}{|x-1|}$$

4. (a) (5 points) Suppose $f(x)$ is the function defined below :

$$f(x) = \begin{cases} 2x+1, & \text{if } x \leq 2 \\ x^2 - 2x + b, & \text{if } x > 2 \end{cases}$$

For which values of the constant parameter b is $f(x)$ continuous everywhere ? Remember to justify your answer.

(b) (5 points) Suppose $f(x)$ is the function defined below :

$$f(x) = \begin{cases} 2x+1, & \text{if } x \leq 2 \\ x^2 - 2x + 5, & \text{if } x > 2 \end{cases}$$

According to the definition of derivative, $f'(2) = \lim_{h \to \square} \frac{f(\square) - f(\square)}{h}$

Fill in the blanks and carefully compute this limit showing all your steps.

5.

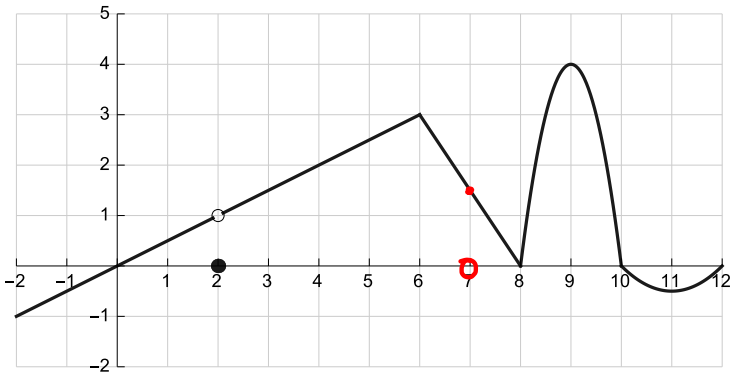
(a) (5 points) Let $g(x) = \sqrt{x} \cdot e^x + x^e + \frac{2}{x} + 3^4$. Compute $g'(x)$. You do not need to simplify your final answer.

(b) (5 points) Find the equation of the line tangent to the curve $y = x^2 + 1$ through the point $P = (1, -\frac{1}{4})$ and $Q(3, 10)$

Find the equations of the lines tangent to $y = x^2 + 1$ and parallel to $y = 4x - 7$

perpendicular to $y = 4x - 7$

4. (12 total points) For this problem, refer to the pictured graph of the function $y = f(x)$ on the interval [-2,12].



(a) (2 points) $\lim_{x\to 7} \dfrac{f(x)-f(7)}{x-7} =$

(b) (2 points) $\lim_{x\to 2} f'(x) =$

(c) (2 points) $\lim_{x\to 2} f''(x) =$

(d) (2 points) $\lim_{x\to 2} \dfrac{f(x)}{x} =$

(e) (2 points) Circle the smallest number in this list:

$$f'(0) \qquad f'(1) \qquad f'(7) \qquad f'(9) \qquad f'(11)$$

(f) (2 points) Give an interval (a,b) on which $f'(x)$ is increasing.

1. Compute each limit. You may use any techniques you know. If a limit is infinite or does not exist, say so, and give a brief explanation why.

   (a) **[5 points]** $\lim_{x \to 2} \dfrac{\sqrt{x^2 + 12}}{(x-2)^2}$

   (b) **[5 points]** $\lim_{x \to \infty} \sqrt{4x^2 + ax + b} - 2x$

   (c) **[5 points]** $\lim_{h \to 0} \dfrac{(x+h)^\pi - x^\pi}{h}$

4. Consider the following function.

$$f(x) = \begin{cases} e^x & \text{if } x < 0 \\ 9x^2 + x + 1 & \text{if } 0 \leq x \leq 2 \\ \dfrac{6x^2 + 13x - 5}{x^2 + x - 6} & \text{if } 2 < x \end{cases}$$

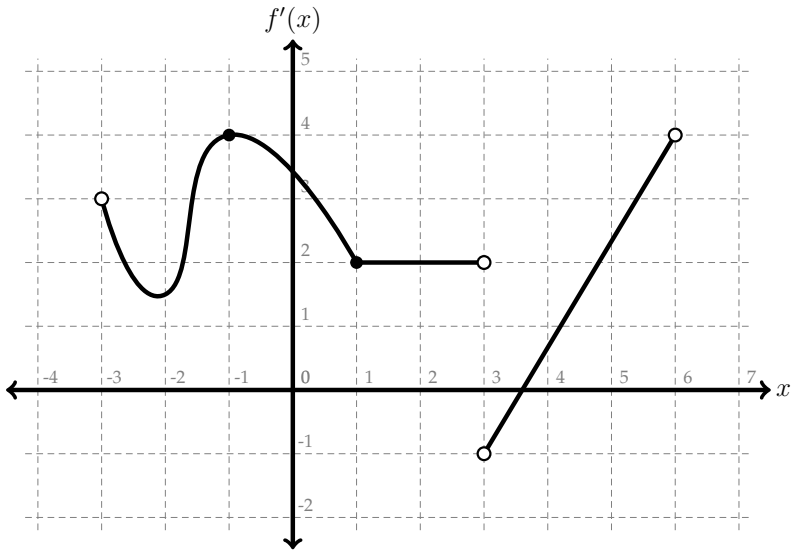
(a) **[5 points]** Is $f(x)$ continuous at $x = 0$? Explain.

(b) **[5 points]** Is $f(x)$ differentiable at $x = 0$? Explain.

(c) **[5 points]** Find all horizontal asymptotes of $f(x)$.

5. This is the graph of $f'(x)$. This is the graph of $f'(x)$. This is the graph of $f'(x)$.

   Sorry to keep repeating myself, but you're going to be really unhappy if you misread the problem. This is the graph of $f'(x)$.



(a) **[3 points]** Find $f''(4)$. Explain.

(b) **[3 points]** Which is greater, $f(1)$ or $f(2)$? Explain.

(c) **[4 points]** Suppose $f(-1) = 2$. Let $h(x) = \dfrac{f(x)}{x^2}$. Find $h'(-1)$.

2. (20 points) Consider this multipart function, where c is a constant:

$$f(x) = \begin{cases} -x + c & \text{if } x \leq 1 \\ 6 - 2x^2 & \text{if } x > 1 \end{cases}$$

(a) (6pts) Find a value of c so that $f(x)$ is continuous at $x = 1$. Explain your reasoning.

(b) (8pts) For the value of c in (a),

$\lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h} =$

$\lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h} =$

(c) (2pts) For the value of c in (a), where is the function $f(x)$ differentiable? Explain.

(d) (4pts) For the value of c in (a), sketch the graph of $y = f'(x)$ below.

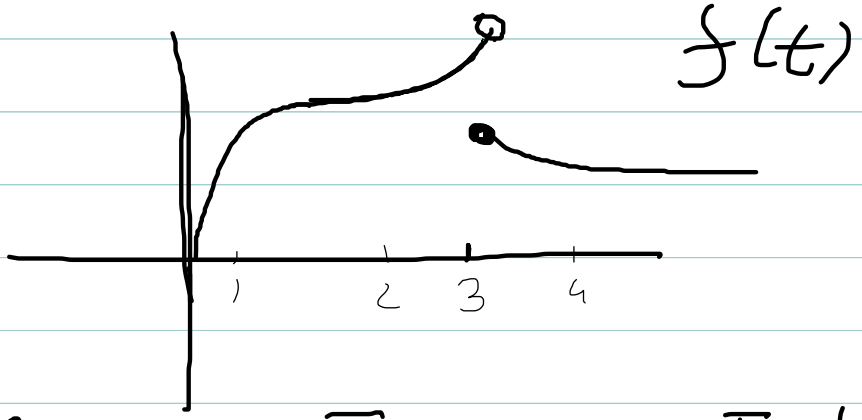
3. (20 points) Consider the curve $y = f(x) = x^2 - 6x + 1$, defined for all values of x.

   (a) (10pts) Find the equation of the tangent line to the curve at the point $(6, 1)$.

   (b) (10pts) Find the coordinates of a point P on the curve where the tangent line has x-intercept 8.

Let $f(t)$ = number of bacteria (in thousands) in a Petri dish at time t (in hours)

Below is the graph of f



Answer True or False

1) $f(t)$ is differentiable at time $t=3$
2) $f'(t)$ is decreasing at $t=4$
3) $f'(1)$ is positive
4) The number of bacteria is increasing faster and faster in the time interval $[2,3]$
5) The number of bacteria is increasing faster at time $t=2$ hours then at $t=1$ hour