1. Compute the derivatives of the following functions. You do not need to simplify.

(a) $f(x) = \sqrt{e^{x^2}+1}$

$$f'(x) = \frac{2x e^{x^2}}{2\sqrt{e^{x^2}+1}}$$

(b) $ln(\ln(\frac{ax^2+b}{cx^5}))$

$$h'(x) = \frac{1}{\ln\left(\frac{ax^2+b}{cx^5}\right)} \cdot \frac{cx^5}{ax^2+b} \quad \frac{2acx^6 + (ax^2+b)5cx^4}{c^2x^{10}}$$

(c) $g(x) = (2x+3)^{\arctan(x)}$

$$y = (2x+3)^{\arctan(x)}$$

$$\ln y = \arctan x \ \ln(2x+3)$$

$$\frac{y'}{y} = \frac{1}{1+x^2}\ln(2x+3) + \frac{\arctan x}{2x+3} \cdot 2$$

$$y' = (2x+3)^{\arctan(x)} \left(\frac{\ln(2x+3)}{1+x^2} + \frac{2\arctan x}{2x+3} \right)$$

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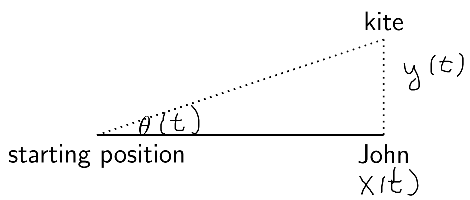
(c) $f(x) = \sqrt{e^{x^2}+1}$

$$f'(x) = \frac{2xe^{x^2}}{2\sqrt{e^{x^2}+1}}$$

2. John starts running in a straight line holding a kite. As he runs he releases the kite's rope so that the kite raises. Assume the kite is always directly above John. When John is 30 feet from his starting position his speed is 5 feet per second and the kite has reached an height (above the ground) of 10 feet and the angle θ formed by the ground and the line joining John 's starting position and the kite (see picture below) is increasing at a rate of 0.2 rad/sec. What is the kite vertical velocity at that time ?



functions $x(t)$, $y(t)$, $\theta(t)$

Know $x(t_1) = 30$, $y(t_1) = 10$

$\theta'(t_1) = 0.2$

$x'(t_1) = 5$

want $y'(t_1)$

$$y(t) = x(t)\tan\theta(t)$$

$$y'(t) = x'(t)\tan\theta(t) + x(t)\sec^2(\theta(t))\,\theta'(t)$$

$$y'(t_1) = 5 \cdot \frac{10}{30} + 30\,\frac{1}{\cos^2(\theta(t_1))} \cdot 0.2$$

Need $\cos\theta(t_1)$

$$\cos\theta(t_1) = \frac{30}{\sqrt{30^2+10^2}}$$

$$y'(t_1) = \frac{5}{3} + 6 \cdot \frac{30^2+10^2}{30^2} = \frac{5}{3} + \cancel{6}^{2} \cdot \frac{10\cancel{00}}{3\cancel{900}} = \frac{25}{3} \text{ feet/sec}$$

3. Let $f(x) = \sin(x) + \cos(x) + \frac{1}{2}$. This problem has two non related parts.

(a) Use linear approximation to approximate $f(0.1)$

$a = 0$, $f(0) = \frac{3}{2}$

$f'(x) = \cos x - \sin x$ $\qquad f'(0) = 1$

$g(x) = \frac{3}{2} + x$

$g(0.1) = 1.6$

(b) The equation $f(x) = 1.8$ has a solution close to $x = \frac{\pi}{3}$. Use linear approximation to approximate this solution.

$a = \pi/3$ $\qquad f(\frac{\pi}{3}) = \frac{\sqrt{3}}{2} + 1$, $f'(\frac{\pi}{3}) = \frac{1}{2} - \frac{\sqrt{3}}{2}$

$g(x) = \frac{\sqrt{3}}{2} + 1 + \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)\left(x - \frac{\pi}{3}\right)$

Solve $g(x) = 1.8$

$\frac{\sqrt{3}}{2} + 1 + \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)\left(x - \frac{\pi}{3}\right) = 1.8$

$x = \frac{\pi}{3} + \dfrac{0.8 - \sqrt{3}/2}{\frac{1}{2} - \frac{\sqrt{3}}{2}} \approx 1.23$

4. The point $P(1,2)$ is on the curve C given by the equation $x^3 - \frac{5}{2}xy + y^2 = 0$

(a) Is C increasing or decreasing at P ? Justify your answer.

$$3x^2 - \frac{5}{2}y - \frac{5}{2}xy' + 2yy' = 0$$

$x = 1 \quad y = 2$

$$3 - 5 - \frac{5}{2}y' + 4y' = 0$$

$$\frac{3}{2}y' = 2 \qquad y' = \frac{4}{3} > 0$$

$\boxed{\text{Increasing}}$

(b) Is C concave up or down at P ? Justify your answer.

$$3x^2 - \frac{5}{2}y - \frac{5}{2}xy' + 2yy' = 0$$

$$6x - \frac{5}{2}y' - \frac{5}{2}y' - \frac{5}{2}xy'' + 2y' \cdot y' + 2yy'' = 0$$

$x = 1 \quad y = 2 \quad y' = \frac{4}{3}$

$$6 - 5 \cdot \frac{4}{3} - \frac{5}{2}y'' + 2 \cdot \frac{16}{9} + 2 \cdot 2y'' = 0$$

$$\frac{3}{2}y'' = \frac{-54 + 60 - 32}{9} < 0$$

$\boxed{\text{Concave down}}$

5. Find the equation(s) of all tangent lines to the curve
$x(t) = t^2 + 3t$, $y(t) = t^2 + 2t + 2 \quad -\infty < t < \infty$, that pass through the point $Q(0, 1)$

$P\,(t^2+3t,\ t^2+2t+2)$ point of tangency

slope of tangent $m = \dfrac{2t+2}{2t+3} = \dfrac{t^2+2t+2-1}{t^2+3t}$

$(2t+2)(t^2+3t) = (2t+3)(t^2+2t+1)$

$\cancel{2t^3} + 6t^2 + 2t^2 + 6t = \cancel{2t^3} + 4t^2 + 2t + 3t^2 + 6t + 3$

$t^2 - 2t - 3 = 0$, $t = -1, 3$

$P_1(-2, 1)$ $\qquad$ $P_2(18, 17)$ (Not needed)

$t=-1$ $\quad m_1 = 0$ $\qquad$ $t=3$ $\quad m_2 = \dfrac{8}{9}$

$\boxed{y = 1}$ $\qquad$ $\boxed{y = 1 + \frac{8}{9}x}$

or $y = 17 + \frac{8}{9}(x - 18)$