Math 124 Section D (Pezzoli)
Winter 2018
Midterm #1

Name _____

TA: _____

. Section: _____

- Your exam contains 6 problems. The entire exam is worth 60 points.
- You have 80 minutes to complete this exam.
- This exam is closed book. You may use one $8\frac{1}{2}" \times 11"$ sheet of notes (both sides). Do not share notes.
- The only calculator allowed is the TI 30x IIS.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 6 pages, including this cover sheet. Please make sure that your exam is complete.

Problem #1(12 pts) _____

Problem #2(15 pts) _____

Problem #3(5 pts) _____

Problem #4(8 pts) _____

Problem #5(5 pts) _____

Problem #6(15 pts) _____

TOTAL (60 pts) _____

1. Calculate the following limits: if the limit exists and it has a finite value, find the value, otherwise decide whether the limit does not exist (DNE) or it is $+\infty$ or $-\infty$. Make sure to justify all steps.

(a) $\lim_{x\to+\infty} \frac{\sqrt{x^2}-4}{8+9x} = \frac{1}{9}$ dominant term circled

or $\lim_{x\to+\infty} \frac{\sqrt{x^2(1-4/x^2)}}{x\left(\frac{8}{x}+9\right)} = \frac{1}{9}$

(b) $\lim_{x\to 2^+} \frac{\sqrt{x^2-4}}{\sqrt{x+7}-3} \cdot \frac{\sqrt{x+7}+3}{\sqrt{x+7}+3} = \frac{\sqrt{x-2}\sqrt{x+2}\,(\sqrt{x+7}+3)}{x+7-9}$

$\lim_{x\to 2^+} \frac{\sqrt{x+2}\,(\sqrt{x+7}+3)}{\sqrt{x-2}} = +\infty$ ($c>0$ in the numerator, $\to 0^+$ in the denominator)

(c) $\lim_{t\to\infty} \frac{1}{t^2}\sin(3t)$

$-\frac{1}{t^2} \le \frac{1}{t^2}\sin(3t) \le \frac{1}{t^2}$

$\lim_{t\to+\infty} -\frac{1}{t^2} = \lim_{t\to+\infty} \frac{1}{t^2} = 0$ therefore by the squeeze th

$\lim_{t\to+\infty} \frac{1}{t^2}\sin(3t) = 0$

2. Given the function

$$f(x) = \begin{cases} mx + 1 & \text{if } x \leq 3 \\ x^2 - 2 & \text{if } x > 3 \end{cases}$$

(a) Complete the definition below .
f is continuous at $x = 3$ when $f(3) = \lim_{x \to 3} f(x)$

or $f(3) = \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

(b) For which value(s) of m is f continuous at 3 ? Justify your answer.

$f(3) = 3m+1$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} mx+1 = 3m+1$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x^2 - 2 = 7$

We need $3m+1 = 7$ $\boxed{m = 2}$

For the next three questions assume $m = 2$.

(c) Write down the limit definition of $f'(3)$ Note: we saw two definitions, you can choose the one you like.

$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$ or $\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$

(d) Compute the limit that you have written in part (c).

$\lim_{h \to 0^-} \frac{2(3+h) + 1 - 7}{h} = 2$

$\lim_{h \to 0^+} \frac{(3+h)^2 - 2 - 7}{h} = \frac{9 + 6h + h^2 - 9}{h} = 6$

so $\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$ DNE

(e) Is f differentiable at $x = 3$?

No $f'(3)$ DNE

3. The tangent to the curve $y = x^2 + ax - 5$ at $P(1, a-4)$ is parallel to the line y=3x+1. Find a .

$f'(x) = 2x + a$

$f'(1) = 2 + a$

$2 + a = 3$

$\boxed{a = 1}$

4. Calculate the derivatives of the following functions,you do not need to simplify your result:

   a) $f(x) = 3e^x x^6 \cos x$

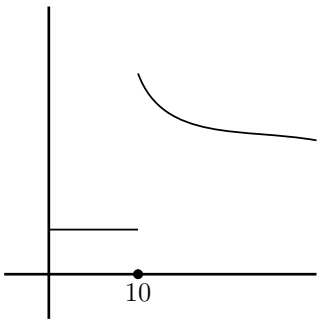
$f'(x) = (3e^x x^6 + 18e^x x^5)\cos x - 3e^x x^6 \sin x$

   b) $g(x) = 5\sqrt{x} + 2e^{\pi} + \dfrac{x^{\sqrt{2}}}{\sin x}$

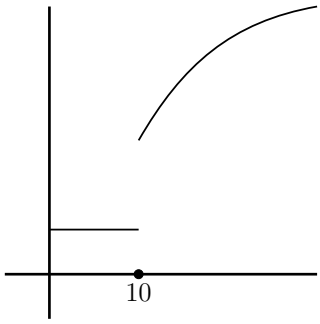
$g'(x) = \dfrac{5}{2\sqrt{x}} + \dfrac{\sqrt{2}x^{\sqrt{2}-1}\sin x - x^{\sqrt{2}}\cdot\cos x}{\sin^2 x}$

5. In this problem salt is measured in grams and time in minutes. The amount $f(t)$ of salt in a solution is 2 g at time $t = 0$; in the interval from $t = 0$ to $t = 10$ salt is added to the solution at a constant rate of c g/min,then at time $t = 10$, 7 g of salt are poured in all at once and after that salt is added again at a faster and faster rate, that is the rate at which salt is being added increases with time after time $t = 10$.
Which of the following graphs could be the graph of $f(t)$? Circle the correct graph.

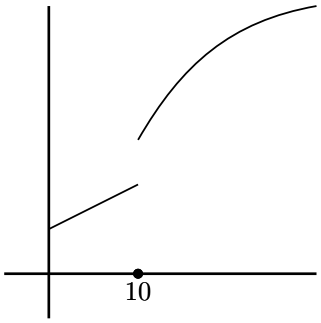
A



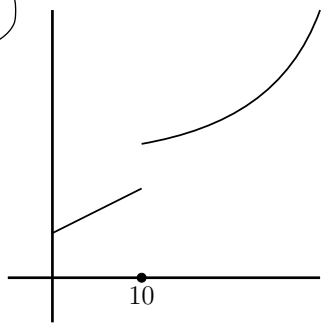
B



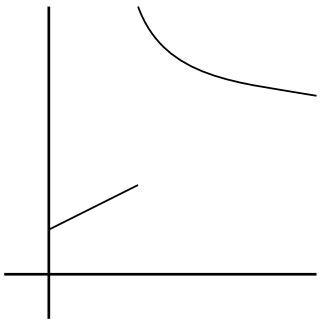
C



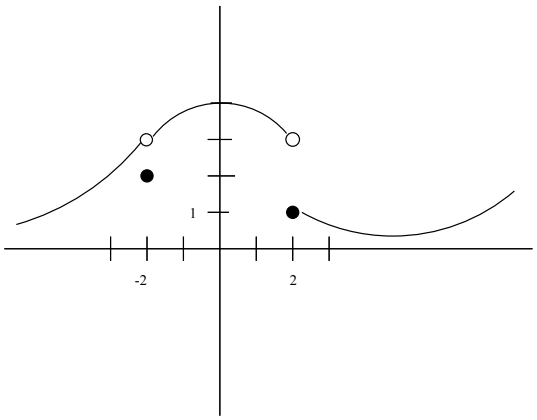
D



E



6. Below is the graph of a function $y = f(x)$. Compute (no justification necessary): (If a limit does not exist write DNE)



(a) $\lim_{x\to 2} f(x) =$ DNE

(b) $\lim_{x\to 0} f(x) =$ 4

(c) $\lim_{h\to 0} \frac{f(h)-4}{h} =$ $f'(0) = 0$

(d) $\lim_{x\to -2} f(x) =$ 3

(e) $\lim_{x\to 0} \frac{f(x)}{x} =$ DNE