Math 124 Section C (Pezzoli)
Winter 2018
Midterm #1

Name _____

TA: _____

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Section: _____

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- Your exam contains 6 problems. The entire exam is worth 60 points.
- You have 80 minutes to complete this exam.
- This exam is closed book. You may use one $8\frac{1}{2}" \times 11"$ sheet of notes (both sides). Do not share notes.
- The only calculator allowed is the TI 30x IIS.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 6 pages, including this cover sheet. Please make sure that your exam is complete.

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Problem #1(12 pts) _____

Problem #2(15 pts) _____

Problem #3(5 pts) _____

Problem #4(8 pts) _____

Problem #5(5 pts) _____

Problem #6(15 pts) _____

TOTAL (60 pts) _____

12 points

1. (~~15 points~~) Calculate the following limits: if the limit exists and it has a finite value, find the value, otherwise decide whether the limit does not exist ( DNE) or it is $+\infty$ or $-\infty$. Make sure to justify all steps.

5 pt (a) $\lim_{x\to 3^+} \dfrac{\sqrt{x^2-9}}{\sqrt{x+13}-4}$

2 pt for rationalizing

$\cdot \dfrac{\sqrt{x+13}+4}{\sqrt{x+13}+4} = \dfrac{\sqrt{x-3}\sqrt{x+3}\left(\sqrt{x+13}+4\right)}{x+13-16} =$

1 pt for algebra

$= \lim_{x\to 3^+} \dfrac{\sqrt{x+3}\left(\sqrt{x+13}+4\right)}{\sqrt{x-3}}$ $\quad -b<0$ $\quad \to 0^+$

2 pt final answer

$= +\infty$

3 pt (b) $\lim_{x\to+\infty} \dfrac{\sqrt{x^2}-9}{2x+13} = \dfrac{1}{2}$ Dominant term circled

or $\lim_{x\to+\infty} \dfrac{\sqrt{x^2(1-9/x^2)}}{x\left(2+\frac{13}{x}\right)} = \dfrac{1}{2}$

4 pt (c) $\lim_{t\to\infty} \dfrac{1}{t^2}\sin(2t)$

$-\dfrac{1}{t^2} \le \dfrac{1}{t^2}\sin(2t) \le \dfrac{1}{t^2}$

$\lim_{t\to+\infty} -\dfrac{1}{t^2} = \lim_{t\to+\infty} \dfrac{1}{t^2} = 0$

Therefore by the squeeze th. $\lim_{t\to+\infty} \dfrac{1}{t^2}\sin(2t) = 0$

2. (~~20 points~~) Given the function

$$f(x) = \begin{cases} 2mx - 3 & \text{if } x \leq 1 \\ x^2 - 2 & \text{if } x > 1 \end{cases}$$

2 pt (a) Complete the definition below .
$f$ is continuous at $x = 1$ when $f(1) = \lim_{x \to 1} f(x)$

or $f(1) = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x)$

3 pt (b) For which value(s) of $m$ is $f$ continuous at 1 ? Justify your answer.

$f(1) = 2m - 3$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 2mx - 3 = 2m - 3$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 - 2 = -1$

we need $2m - 3 = -1$ $\boxed{m = 1}$

For the next three questions assume $m = 1$.

2pt (c) Write down the limit definition of $f'(1)$ Note: we saw two definitions, you can choose the one you like.

$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$ or $\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$

(d) Compute the limit that you have written in part (c).

4 pt $\lim_{h \to 0^-} \frac{2(1+h) - 3 - (-1)}{h} = 2$

$\lim_{h \to 0^+} \frac{(1+h)^2 - 2 - (-1)}{h} = \frac{1 + 2h + h^2 - 1}{h} = 2$

so $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = 2$

1pt (e) Is $f$ differentiable at $x = 1$ ?

yes $f'(1) = 2$

3. (10 ~~points~~) The tangent to the curve $y = x^2 + ax + 5$ at $P(1, 6 + a)$ is parallel to the line $y = 4x + 2$. Find $a$

$f'(x) = 2x + a$ 2 pt

$f'(1) = 2 + a$ 2 pt

$2 + a = 4$ so $\boxed{a = 2}$ 1 pt

4. (~~10~~ ~~points~~) Calculate the derivatives of the following functions,you do not need to simplify your result:

a) $f(x) = 5e^x x^2 \sin x$ 4 pt

$f'(x) = (5e^x \cdot x^2 + 10e^x \cdot x)\sin x + 5e^x \cdot x^2 \cos x$

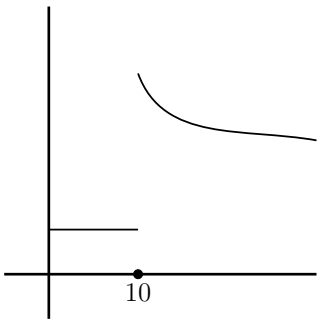
b) $g(x) = 3\sqrt{x} + 2e^{\pi} + \dfrac{\cos x}{x^{\sqrt{2}}}$ 4 pt

$g'(x) = \dfrac{3}{2\sqrt{x}} + \dfrac{-\sin x \cdot x^{\sqrt{2}} - \cos x \cdot \sqrt{2}\, x^{\sqrt{2}-1}}{x^{2\sqrt{2}}}$

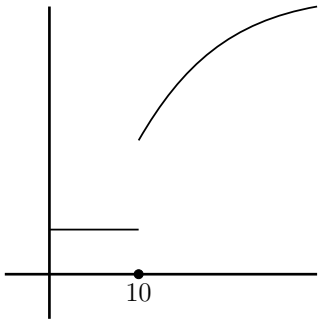
4 pt

5. (~~10 points~~) In this problem salt is measured in grams and time in minutes. The amount $f(t)$ of salt in a solution is 2 g at time $t = 0$; in the interval from $t = 0$ to $t = 10$ salt is added to the solution at a constant rate of $c$ g/min,then at time $t = 10$ , 7 g of salt are poured in all at once and after that salt is added again at a slower and slower rate, that is the rate at which salt is being added decreases with time after time $t = 10$. Which of the following graphs could be the graph of $f(t)$ ? Circle the correct graph.

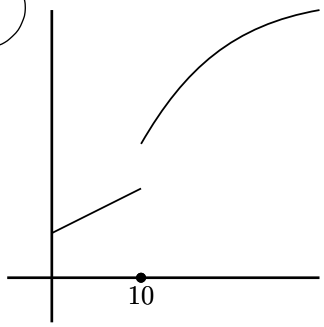
A



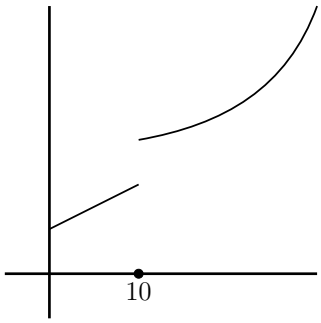

B



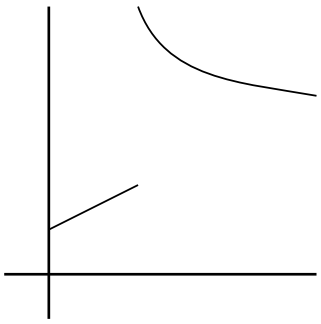

C



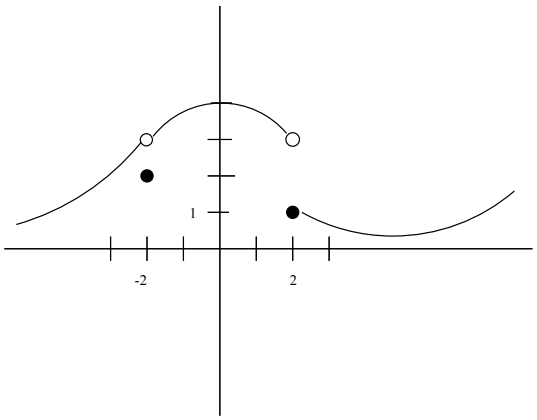

D




E



6. (15 points) Below is the graph of a function $y = f(x)$. Compute (no justification necessary): (If a limit does not exist write DNE)




(a) $\lim_{x\to 0} f(x) =$ 4 1pt

(b) $\lim_{x\to 2} f(x) =$ DNE 2pt

(c) $\lim_{x\to -2} f(x) =$ 3 2pt

(d) $\lim_{x\to 0} \frac{f(x)}{x} =$ DNE 2pt

(e) $\lim_{h\to 0} \frac{f(h)-4}{h} =$ $f'(0) = 0$ 2pt