2.7

The derivative of f(x) at a point x0

f' (x0)

$(x_0, f(x_0))$

Given any function f(x) and point xo on the x axis we can calculate

$f'(x_0)$, $\frac{df}{dx}(x_0)$

$$\lim_{x \to xo} \frac{f(x) - f(x0)}{x - x0} \qquad \frac{\Delta y}{\Delta x}$$

If this limit exists and it is finite, we will say f is differentiable at x0 and use the following notation

To denote the derivative of f at x0. (note :it is a number , what are its units?)

$$\frac{\text{units for } f}{\text{units for } x}$$

$$f'(x0)$$

$$\frac{df}{dx}(x0)$$

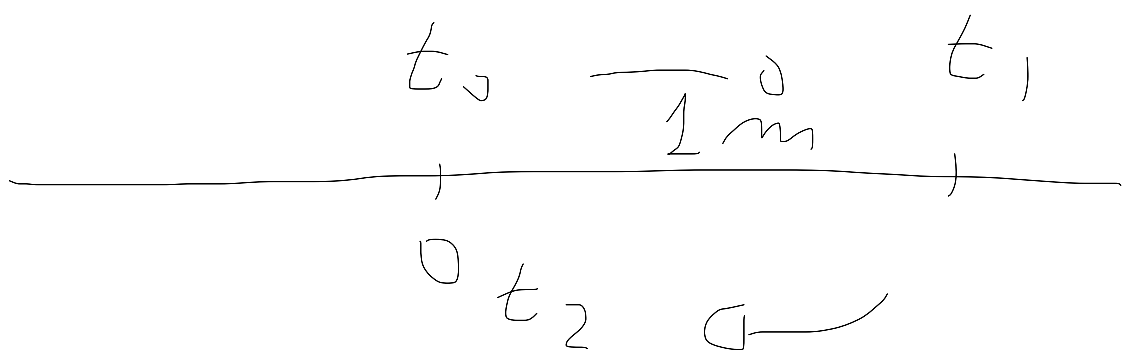
$f'(x_0)$ gives us

- Slope of tangent at x_0, through $P(x_0, f(x_0))$
- Rate of change at x_0
- velocity

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$v(t0) = \lim_{t1 \to t0} \frac{s(t1) - s(t0)}{t1 - t0} = s^{'}(t0)$$

Where s(t) is the position function .
Book also uses d(t) instead of s(t).
Discuss difference between position and distance travelled.



Equivalent limit definition

- x =x0+h

$f'(x_0) =$

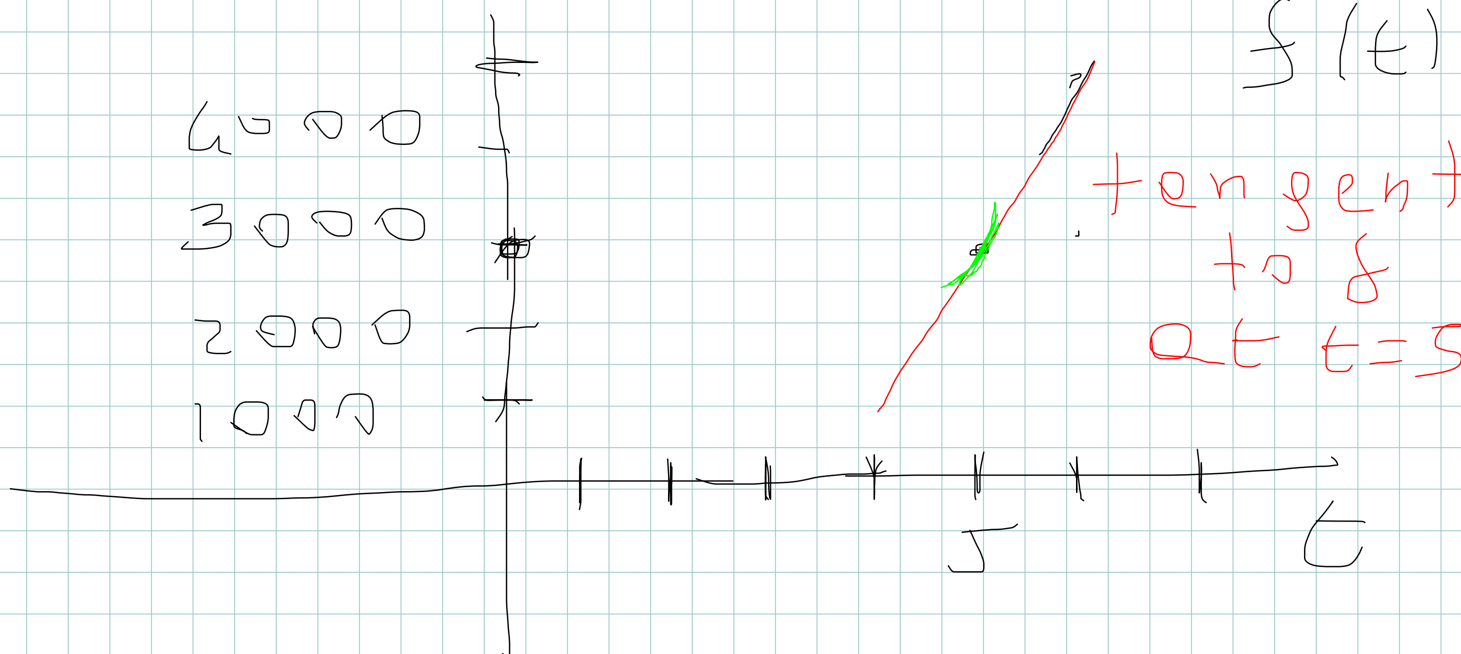
$$\lim_{x \to xo} \frac{f(x) - f(x0)}{x - x0} \quad = \lim_{h \to 0} \frac{f(x0 + h) - f(x0)}{h}$$

- The number of bacteria present in a Petri dish t hours after the start of an experiment is given by the function y=f(t).
- What is the meaning of f'(5) ? rate at which # bacteria is is changing at $t=5$
- What are the units of f' (5) ? # bacteria/hour
- What does f'(5)=2000 tell you ?
- What does f'(6)=0 tell you ? rate of change is $\varnothing$
- What does the sign of f'(5) tell you ?

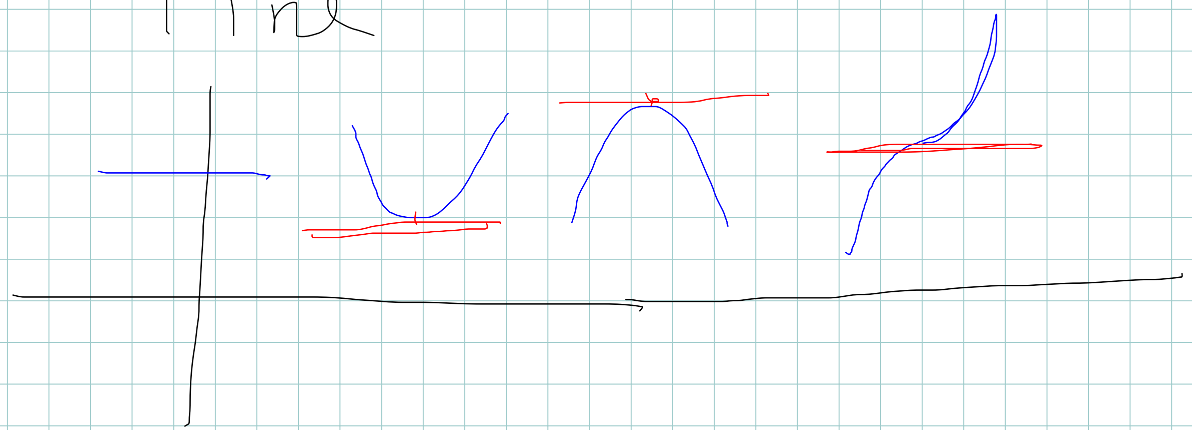
wether f(t) is increasing ($f'(5)>0$) or decreasing ($f'(5)<0$)

$f'(5) = 2000$

At $t=5$ number of bacteria in Petri dish is increasing at a rate of 2000 bacteria/hour



Horizontal tangent line



More of this in ch 4

Using the definition of derivative calculate

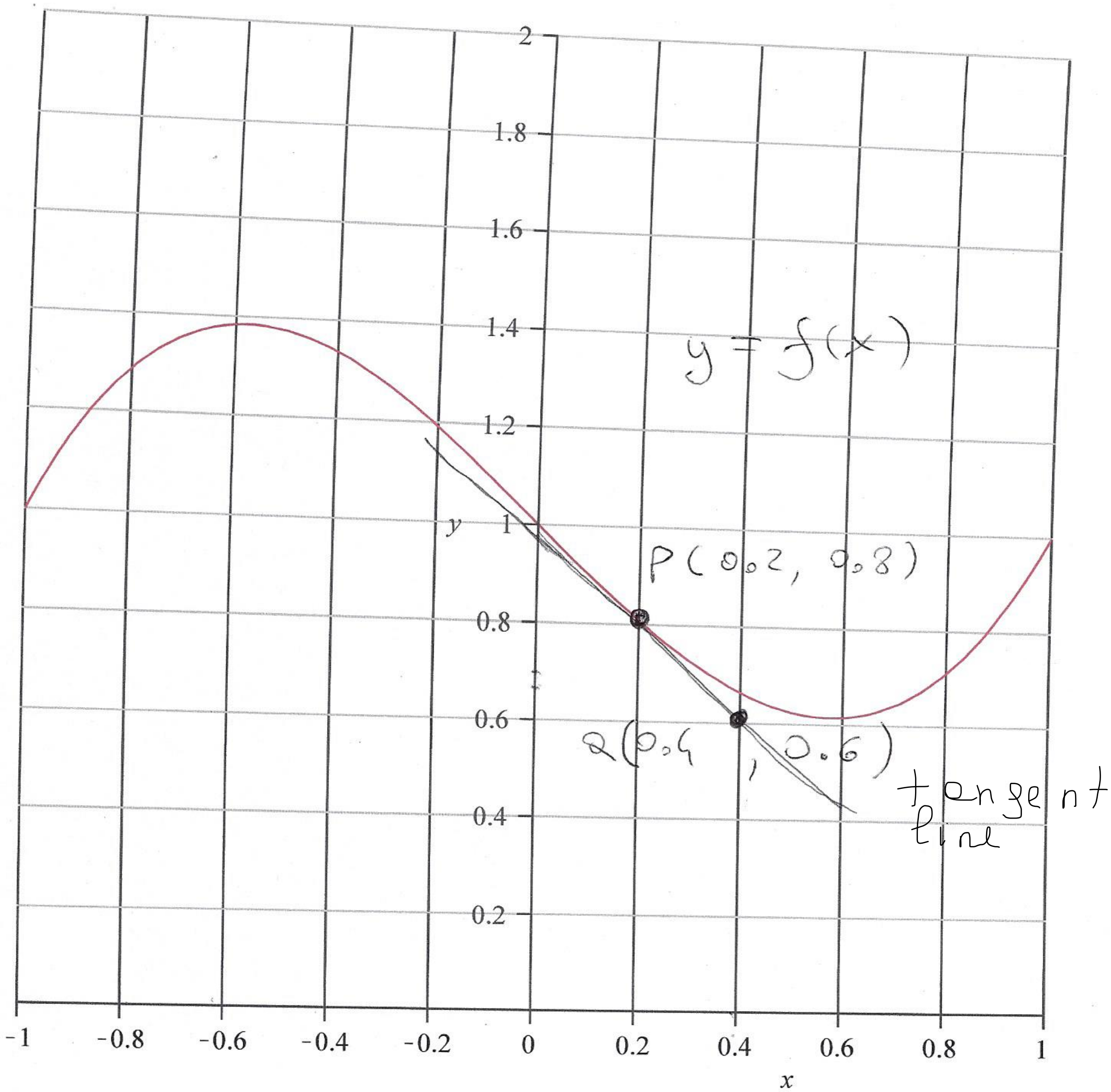
- f’(1) , where f(x)= x^2 +2.

$$\lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \frac{(1+h)^2 + 2 - (1^2+2)}{h}$$

$$= \frac{(1+h)^2 - 1}{h} = \frac{\cancel{1} + h^2 + 2h \cancel{- 1}}{h} = h + 2$$

$$\to 2$$

$$f'(1) = 2$$



Estimate $f'(0.2)$ = slope tangent line

$\frac{\Delta y}{\Delta x} = -\frac{0.2}{0.2} = -1$. Find the equation of tangent line

$x_0 = 0.2$

~~$y = -x + b$~~ ~~$y = -x + 1$~~

~~$0.8 = -0.2 + b$~~ ~~$b = 1$~~ $y = 0.8 - (x - 0.2)$

Note: if f is differentiable at x_0 then f is continuous at x_0

why ?

$$f(x) \approx \overbrace{f(x_0) + f'(x_0)(x - x_0)}^{\text{equation of tangent line}}$$

so $\displaystyle\lim_{x \to x_0} f(x) = f(x_0)$

$$\lim_{x \to x_0} f(x) = \lim_{x \to x_0} \frac{f(x) - f(x_0) + f(x_0)}{x - x_0}(x - x_0)$$

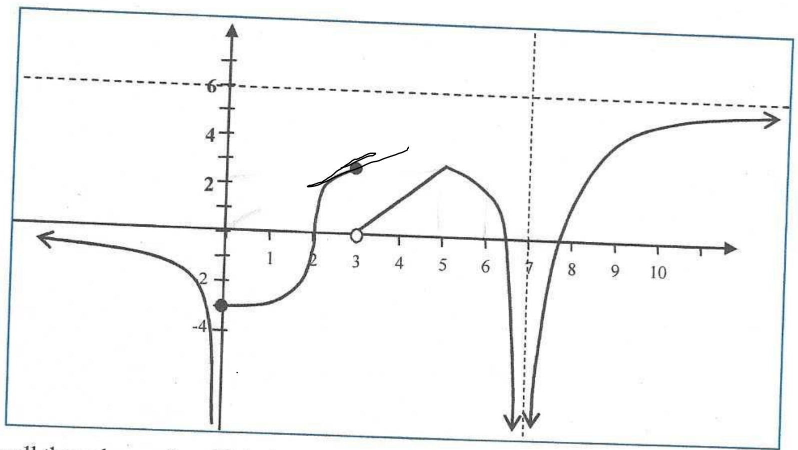
$$= \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}(x - x_0) + f(x_0) = f(x_0)$$

Graphically

f is not dfferentiable at x0 if

- f is not continuous at x0
- The graph of f has a corner at x0
- The graph of f is vertical at x0 (tangent line is vertical). infinite slope
- Problem from Win09 mid1 (Nichifor)

$\boxed{2}$ (12 pts=3+3+2+4) Below is the graph of a function, $y = f(x)$. Use it to answer the following questions, no justification needed:



a) State all the values a for which $f(x)$ is not continuous at $x=a$.

0, 3, 7

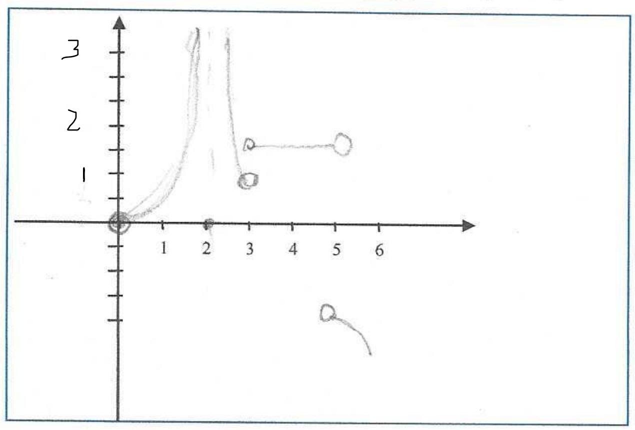
b) State all the values a for which $f(x)$ is not differentiable at $x=a$.

0, 3, 7 2, 5

c) Evaluate the two limits at infinity:

$\lim_{x\to+\infty} f(x) =$ 6 and $\lim_{x\to-\infty} f(x) =$ 0

d) Sketch the portion of the graph of the derivative function $f'(x)$ corresponding to the interval $0 \le x \le 6$.



- Is the following function f(x) differentiable at 0?

$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

$$\lim_{h \to 0} \frac{f(0+h)-f(0)}{h} = \frac{h^2 \sin(\frac{1}{h})}{h}$$

$$\lim_{h \to 0} \overset{0}{h} \cdot \overset{DNE}{\sin\left(\frac{1}{h}\right)}$$

$-1 \le \sin\left(\frac{1}{h}\right) \le 1$ implies

$-h \le h\sin\left(\frac{1}{h}\right) \le h$

if $h > 0$

$\underset{0}{\swarrow} \qquad \searrow 0$

The squeeze th tells me

$$\lim_{h \to 0^+} h\sin\left(\frac{1}{h}\right) = 0$$

$-1 \le \sin\frac{1}{h} \le 1$ implies

$-h \ge h\sin\frac{1}{h} \ge h$

$\to 0$ $\qquad\qquad$ $\to 0$

if $h<0$

The squeeze th tells me

$$\lim_{h\to 0^-} h\sin\frac{1}{h} = 0$$

Therefore $\lim_{h\to 0} h\sin\frac{1}{h} = 0 = f'(0)$

yes differentiable at 0

Tangent line problem

- Given y=f(x) and P=(x0,y0) on the graph of f the line tangent to the graph of f(x) at P has slope f ‘ (x0) and equation:

$$y_0 = f(x_0)$$

$$y\text{-}y0 = f'(x0)(x\text{-}x0)$$

or

$$y = \underbrace{f(x_0)}_{= y_0} + f'(x_0)(x - x_0)$$

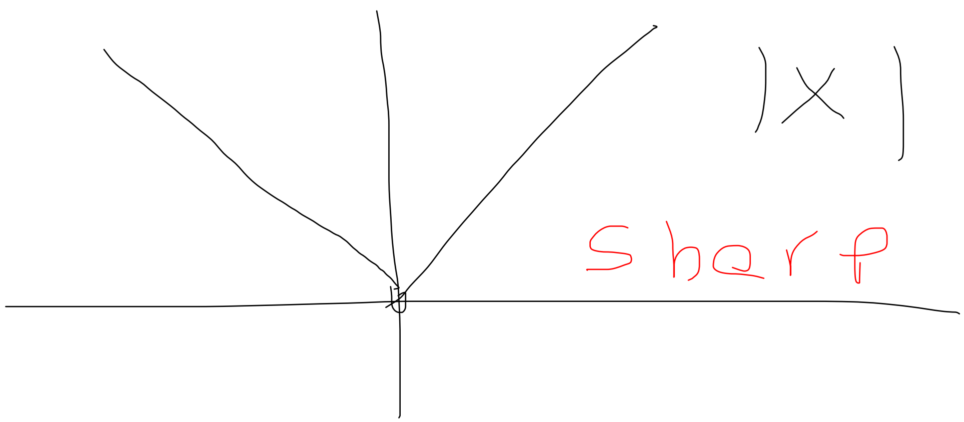
Tangent line Problem (P not on curve)

- Given $y=f(x)$ and $P=(x0,y0)$ NOT on the graph of f ,to find the line(s) tangent to the graph of f(x) passing through P:
- Call $Q(x,y)=(x,f(x))$ the unknown point of tangency
- Write the slope m of the tangent line in two different ways and set them equal

$$m = \frac{f(x) - y0}{x - x0} = f'(x)$$

- Solve for x; for any solution x1, you have a corresponding tangent line
- y= f(x1)+f'(x1)(x-x1)

- Is f(x)=|x| differentiable at 0 ? Graphical interpretation and calculations



$|x|$

Sharp corner

NOT differentiable at 0