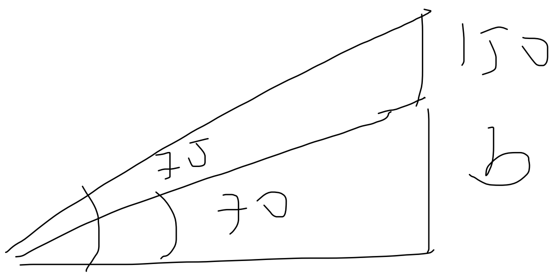
Spring 2014 final

6. **[12 points]** You are standing on flat ground some distance away from a skyscraper. Climbing up the skyscraper, 150 feet from the top, is a gorilla.

   From where you stand, you measure the angle of elevation from the ground to the gorilla, and you find it to be $70°$.

   Then you measure the angle of elevation from the ground to the top of the skyscraper. It's $75°$.

   How tall is the skyscraper?



$$\begin{cases} \tan\left(\frac{70 \cdot 2\pi}{360}\right) = 2.7475 = \frac{b}{a} \\ \tan\left(\frac{75 \cdot 2\pi}{360}\right) = 3.7321 = \frac{b+150}{a} \end{cases}$$

$$\begin{cases} b = 2.7475\, a \\ 3.7321\, a = 2.7475\, a + 150 \end{cases}$$

$$\begin{cases} a = \frac{150}{0.9846} \approx 152.3461 \\ b = 568.57 \text{ feet} \end{cases}$$

8. [12 points] The predicted times and heights of the high and low tides for the seaside village of Portwenn during a certain day are:

Time of day	Low/High Tide Height (in meters)
00:30	4.8
06:30	14.4
12:30	4.8
18:30	14.4

12 hrs apart

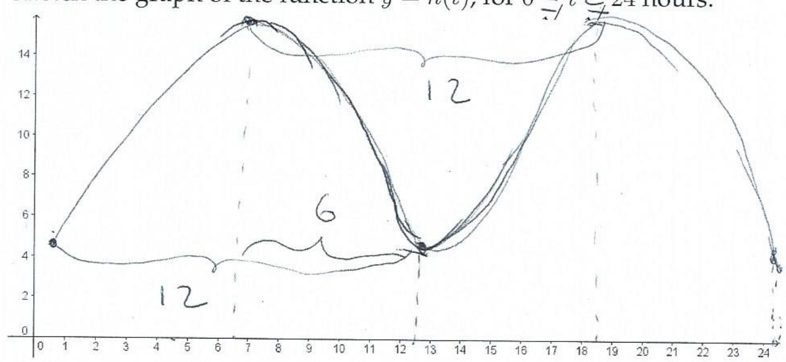
(a) Find a sinusoidal function in standard form, $h(t) = A\sin(\frac{2\pi}{B}(t-C)) + D$, which models the tide height data for Portwenn at t hours past midnight, on the given day.

$A = \frac{14.4 - 4.8}{2} = 4.8$ $\quad D = \frac{14.4 + 4.8}{2} = 9.6$

$B = 12$ $\quad C = 6.5 - \frac{12}{4} = 3.5$

$h(t) = 4.8 \sin\left(\frac{2\pi}{12}(t - 3.5)\right) + 9.6$

(b) Sketch the graph of the function $y = h(t)$, for $0 \leq t \leq 24$ hours.



(c) A boat requires a tide height of 10 meters or more to be able to enter a harbor. Compute all the time intervals during this day when the boat could enter the Portwenn harbor.

Solve $4.8 \sin\left(\frac{2\pi}{12}(t-3.5)\right) + 9.6 = 10 \quad 0 \leq t \leq 24$

$\sin\left(\frac{\pi}{6}(t - 3.5)\right) = \frac{10 - 9.6}{4.8} = \frac{1}{12}$

1) principal solution: $\frac{\pi}{6}(t - 3.5) = \sin^{-1}(\frac{1}{12})$

$t = \frac{6}{\pi}\sin^{-1}\left(\frac{1}{12}\right) + 3.5 \approx 3.66$

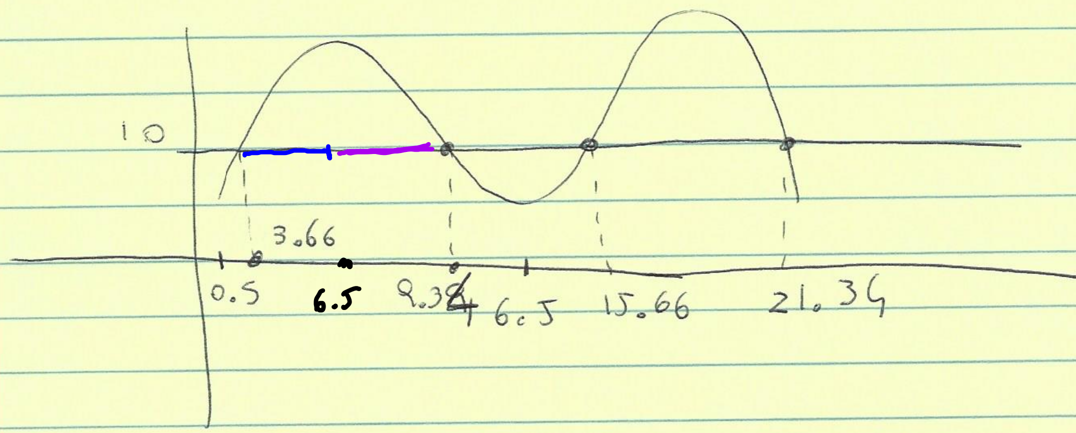
other solutions $3.66 + 12 = 15.66$

2) Symmetric solution

$\frac{\pi}{6}(t - 3.5) = \pi - \sin^{-1}\left(\frac{1}{12}\right)$

$t = \frac{6}{\pi}\left(\pi - \sin^{-1}\left(\frac{1}{12}\right)\right) + 3.5 \approx 9.34$

other solution $\quad 9.34 + 12 = 21.34$



The boat can enter

from $t = 3.66$ to $t = 9.34$

and from $t = 15.66$ to $t = 21.34$

Other ways to calculate symmetric solution

Look at graph : $t = 6.5 + (6.5 - 3.66) = 9.34$

or

Use formula : $2C + \frac{B}{2} - \text{principal} = 2 \cdot 3.5 + \frac{12}{2} - 3.66$

$= 7 + 6 - 3.66 = 9.34$