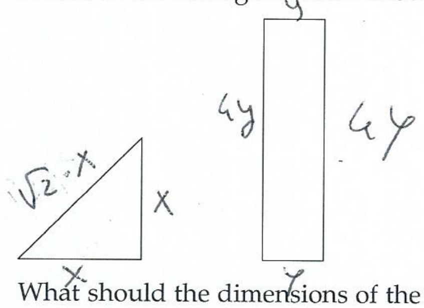
Conroy Fall 2013

3. You have 1000 meters of fencing with which to build two enclosures. One enclosure will be an isosceles right triangle, and the other will be a rectangle that is four times as long as it is wide. The figure below shows the two shapes.



What should the dimensions of the rectangular enclosure be to **minimize** the combined total area of the two enclosures?

$$A = \frac{1}{2}x^2 + 4y^2$$

$$1000 = (\sqrt{2}+2)x + 10y \qquad x = \frac{1000-10y}{\sqrt{2}+2}$$

$$A = \frac{1}{2}\left(\frac{1000-10y}{\sqrt{2}+2}\right)^2 + 4y^2 =$$

$$= \left(\frac{50}{(\sqrt{2}+2)^2} + 4\right)y^2 - \frac{10^4}{(\sqrt{2}+2)^2}y + \frac{1}{2}\left(\frac{1000}{\sqrt{2}+2}\right)^2$$

This is a looking up parabola so the minimum is at the vertex, so for

$$y = -\frac{b}{2a} = \frac{\frac{10^4}{(\sqrt{2}+2)^2}}{2\left(\frac{50}{(\sqrt{2}+2)^2}+4\right)} \approx \boxed{51.75}$$

and

$$4y \approx \boxed{206.98}$$

Consider the function $f(x) = 3 - \sqrt{2x-1}$.

1. Find the domain and range of $f(x)$.

for domain $2x-1 \geq 0$ $\quad x \geq \frac{1}{2}$

for range $\sqrt{2x-1}$ can take any value $v \geq 0$

DOMAIN = $[1/2, +\infty)$

RANGE = $(-\infty, 3]$

2. Compute the inverse function $f^{-1}(y)$. Show all steps. Indicate the domain for the inverse function.

$$y = 3 - \sqrt{2x-1}$$

$$\sqrt{2x-1} = 3 - y$$

$$2x-1 = (3-y)^2$$

$$x = \frac{1 + (3-y)^2}{2}$$

$$f^{-1}(y) = \frac{1 + (3-y)^2}{2}$$

Domain $(-\infty, 3]$

Range $[1/2 \; +\infty)$

3. Compute $f(f(1))$

$f(1) = 2 \qquad f(f(1)) = f(2) = 3 - \sqrt{3}$