Ch 8

Composition
\[ z = g(f(x)) \]

\[ E \times f(x) = x^2 \]

\[ g(x) = e^x \]

\[ g(f(x)) = g(x^2) = e^{x^2} \]

\[ f(g(x)) = \int g(x) = \int e^x = (e^x)^2 \]

If \( x = 3 \)

\[ e^3 = e^9 \approx 8103 \]

\[ (e^3)^2 \approx 403 \]
Ex:

\[ f(x) = x^2 + 1 \]
\[ g(x) = 2x + 3 \]

\[ g(f(x)) = g(x^2 + 1) \]
\[ = 2(x^2 + 1) + 3 = 2x^2 + 5 \]

\[ f(g(x)) = f(2x + 3) = \]
\[ = (2x + 3)^2 + 1 = \]
\[ 4x^2 + 12x + 10 \]
\[ E \times \mathcal{S}(x) = \begin{cases} \circ x + 1 & \text{if } x \leq 0 \\ \circ x + x + 1 & \text{if } x > 0 \end{cases} \]

\[ g(x) = 2x - 3 \]

\[ g\left(\mathcal{S}(x)\right) = \begin{cases} 2(x+1) - 3 & \text{if } x \leq 0 \\ 2(2x + x + 1) - 3 & \text{if } x > 0 \end{cases} \]

\[ f\left(g(x)\right) = f(2x - 3) = \begin{cases} (2x - 3) + 1 & \text{if } 2x - 3 \leq 0 \\ 2(2x - 3)^2 + (2x - 3) + 1 & \text{if } 2x - 3 > 0 \end{cases} \]
Write $e^{x^3}$ as the composition of two functions

$f(x) = x^3 \quad g(x) = e^x$

$g(f(x)) = g(x^3) = e^{x^3}$

Write $\sqrt{x^3 + 1}$ as the composition of two functions

$f(x) = x^3 + 1 \quad g(x) = \sqrt{x}$

$g(f(x)) = \sqrt{x^3 + 1}$
\[ f(x) = x + 1 \]
\[ g(x) = \sqrt{x} \]

Can I consider \( g(f(x)) \)?

\[ g(f(x)) = \sqrt{x + 1} \]

is only defined if \( x \geq -1 \), but \( f \) is defined for all \( x \).
General rule for domain of $g(f(x))$

1) $x$ has to be in the domain of $f$

2) $f(x)$ has to be in the domain of $g(x)$

Example:

$E \times f(x) = \sqrt{x} - 1$

$g(x) = \sqrt{x}$

$g(f(x)) = g(\sqrt{x} - 1) =$

$= \sqrt{\sqrt{x} - 1}$
\[ f(x) = \sqrt{x} - 1 \quad g(x) = \sqrt{x} \]

\[ g(f(x)) = \sqrt{\sqrt{x} - 1} \]

**Domain**

\[ \sqrt{x} - 1 \geq 0 \quad x \geq 0 \]

\[ x \geq 1 \quad x \leq -1 \]

**Domain**

\[ x \geq 1 \]
Range of a function

1) Graphically, if graph is available, read on y axis

2) Algebraically

Example \( f(x) = \frac{1}{x-1} \)

Set \( y = \frac{1}{x-1} \)

Solve for \( x \) \( x-1 = \frac{1}{y} \)

\( x = 1 + \frac{1}{y} \)

For which \( y \) does \( y \)
this formula make sense.

If $y \neq 0$

As long as $y \neq 0$, I get a value $x$ in the domain of $f \quad (x \neq 1)$

so $\text{Range} f = y \neq 0$
Suppose domain of \( g(x) \) is \(-5 \leq x \leq 6\).

Range of \( g \) (all values \( g \) takes) is \( 1 \leq y \leq 10 \).

\[ f(x) = 4 \, x - 5 \] defined on \(-\infty < x < \infty\).
for \ g(d(x)) = g(4x - 5) \ \ \ \ (8)

Domain is

is \ -5 \leq 4x - 5 \leq 6 \n
0 \leq x \leq \frac{11}{5}

range is \ 1 \leq y \leq 10
more complicated for general \ f

for \ f(g(x)) = 4g(x) - 5 \ \ \ \ y

Domain is \ -5 \leq x \leq 6

range is: first write
\( 0 \leq g(x) \leq 40 \)

then

\( 0 \leq 4 \cdot g(x) \leq 40 \)

\( -1 \leq 4 \cdot g(x) - 5 \leq 35 \)

\( -1 \leq y \leq 35 \)
\[ f(t) = t - 1, \quad g(t) = |t| \]

\[ g(f(t)) = |t - 1| \]

\[ |t - 1| = \begin{cases} 
  t - 1 & \text{when } t \geq 1 \\
  -(t - 1) & \text{when } t < 1 
\end{cases} \]

If \[ h(t) = |t| \]
find a formula for \[ h(h(t) - 2) \]

\[ h(|t| - 2) = | |t| - 2 | \]
\(
\begin{align*}
\lfloor t \rfloor - 2 &= \\
\begin{cases}
  t - 2 & \text{if } t > 0 \\
  t + 2 & \text{if } t \leq 0
\end{cases}
\end{align*}
\)
Suppose \( f(x) \) is the profit made by selling \( x \) barrels of apples and \( g(x) \) is the number of barrels of apples produced by \( x \) trees. Explain the meaning of \( f(g(x)) \), "profit made by selling the barrel of apples produced by \( x \) trees."