Math 120 (Pezzoli)
Fall 2019
Midterm #2

Name _____

TA: _____

. Section: _____

Instructions:

- Your exam contains 3 problems.
- Your exam should contain 4 pages; please make sure you have a complete exam.
- Box in your final answer.
- Unless stated otherwise, you **MUST** show work for credit. No credit for answers only. If in doubt, ask for clarification.
- Your work needs to be neat and legible.
- You are allowed one 8.5×11 sheet of notes (both sides).
- The only calculator allowed is the Ti-30x IIS.
- Round off your final answers to 2 decimal places, unless you are asked for exact answers.

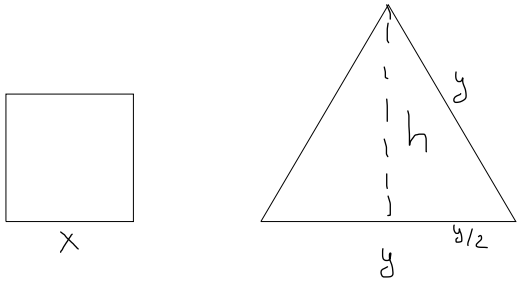
Problem #1 (10 pts) _____

Problem #2 (12 pts) _____

Problem #3 (13 pts) _____

TOTAL (35 pts) _____

1. You want to build two enclosures using exactly 3000 feet of fencing. One enclosure will be an equilateral triangle, the other a square. What should the side of the square be in order to minimize the area of the two combined enclosures ?



$h^2 + \left(\frac{y}{2}\right)^2 = y^2$

$h = \sqrt{y^2 - \frac{y^2}{4}} = y \cdot \frac{\sqrt{3}}{2}$

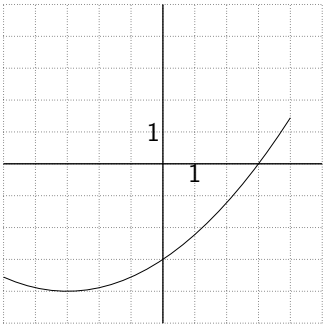
$A = x^2 + \frac{1}{2} y \cdot y \frac{\sqrt{3}}{2}$

$4x + 3y = 3000 \qquad y = 1000 - \frac{4}{3}x$

$A = x^2 + \frac{\sqrt{3}}{4}\left(1000 - \frac{4}{3}x\right)^2 = x^2 + \frac{\sqrt{3}}{4}\left(10^6 - \frac{8000}{3}x + \frac{16}{9}x^2\right) =$

$= \left(1 + \frac{4\sqrt{3}}{9}\right)x^2 - \frac{8000\sqrt{3}}{12}x + \frac{\sqrt{3}}{4}10^6$ The graph of A(x) looks up $\cup$ min is at vertex $h = \frac{\frac{8000\sqrt{3}}{12}}{2\left(1 + \frac{4\sqrt{3}}{9}\right)} \approx 326.22$ m.

2. The function f graphed below has domain $-5 \leq x \leq 4$



a) What is the value of $f(f(0))$?

$f(-3) = -4$

b) What is the domain of $f(\frac{x}{2})$?

$-10 \leq x \leq 8$

The next two questions are unrelated to parts a), b) above. Consider the function $g(x) = 2(x-1)^2 + 4$.

c) Write a formula for the function whose graph is the graph of g shifted horizontally to the right of two units, then reflected across the y axix, then shifted vertically up of three units .

1) $2(x-3)^2+4$
2) $2(-x-3)^2+4$
3) $2(-x-3)^2+7$

d) Let $h(x)$ be the function you obtain by restricting $g(x)$ to the domain $x \leq 0$ Find a formula for $h^{-1}(y)$, the inverse of $h(x)$, and find the domain of $h^{-1}(y)$.

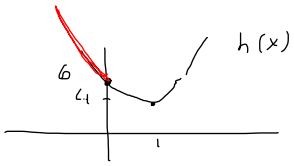
DOMAIN = $[6, +\infty)$

$y = 2(x-1)^2 + 4$

$\frac{y-4}{2} = (x-1)^2$

$1 \pm \sqrt{\frac{y-4}{2}} = x$

$1 - \sqrt{\frac{y-4}{2}} = x$



range h is $[6, +\infty)$

3. John invested $ 1,000 in 2015. Mary invested $ 1,000 in 2016.

   Assume both investments grow exponentially. John 's investment increases 3 % every two years. Mary's doubles every 15 years. When will Mary have three times as much money invested as John ? Give the answers in years (Ex: in the year 2040)

$t = 0$ corresponds to 2015

$f(t) = 1000 \cdot \left(\sqrt{1.03}\right)^t$ value of John's investment

$g(t) = 1000 \left(\sqrt[15]{2}\right)^{t-1}$ value of Mary's investment

Want: $g(t) = 3 f(t)$

$$1000 \left(\sqrt[15]{2}\right)^{t-1} = 3 \cdot 1000 \left(\sqrt{1.03}\right)^t$$

$$\left(\frac{\sqrt[15]{2}}{\sqrt{1.03}}\right)^t = 3 \sqrt[15]{2}$$

$$t = \frac{\ln\left(3\sqrt[15]{2}\right)}{\ln\left(\frac{\sqrt[15]{2}}{\sqrt{1.03}}\right)} \approx 36$$

In 2051

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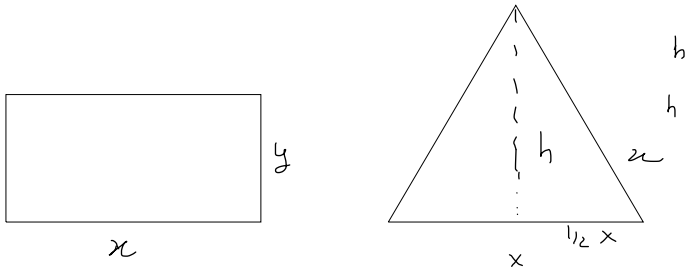
Problem #1 (10 pts) _____

Problem #2 (12 pts) _____

Problem #3 (13 pts) _____

TOTAL (35 pts) _____

1. You want to build two enclosures using exactly 3000 feet of fencing. One enclosure will be an equilateral triangle, the other a rectangle. The basis of the rectangle has the same length as the sides of the triangle. What should the side of the triangle be in order to maximize the area of the two combined enclosures ?



$h^2 + \left(\frac{1}{2}x\right)^2 = x^2$

$h = \sqrt{x^2 - \frac{x^2}{4}} = \frac{x}{2}\sqrt{3}$

$A = xy + \frac{1}{2} x \cdot \frac{x}{2}\sqrt{3}$

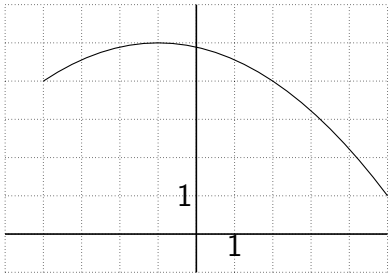
$5x + 2y = 3000 \qquad y = 1500 - \frac{5}{2}x$

$A = x\left(1500 - \frac{5}{2}x\right) + \frac{\sqrt{3}}{4}x^2 = \left(\frac{\sqrt{3}}{4} - \frac{5}{2}\right)x^2 + 1500x$

≈ -2.07

The graph of $A(x)$ looks like $\cap$ so max is at the vertex $h = \frac{-1500}{2\left(\frac{\sqrt{3}}{4} - \frac{5}{2}\right)} = \boxed{362.85}$

2. The function f graphed below has domain $-4 \leq x \leq 5$



a) What is the value of $f(f(-1))$?

$f(5) = 1$

b) What is the domain of $f(2x)$?

$-\frac{4}{2} \leq x \leq \frac{5}{2}$

The next two questions are unrelated to parts a), b) above. Consider the function $g(x) = 3(x+1)^2 - 5$.

c) Write a formula for the function whose graph is the graph of g shifted horizontally to the left of two units, then shifted across the x axis, then shifted vertically down of three units .

1) $3(x+3)^2 - 5$

2) $-3(x+3)^2 + 5$

3) $-3(x+3)^2 + 2$

d) Let $h(x)$ be the function you obtain by restricting $g(x)$ to the domain $x \geq 0$ Find a formula for $h^{-1}(y)$, the inverse of $h(x)$, and find the domain of $h^{-1}(y)$.

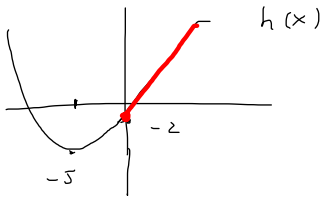
DOMAIN = $[-2, +\infty)$

$y = 3(x+1)^2 - 5$

$\frac{y+5}{3} = (x+1)^2$

$-1 \pm \sqrt{\frac{y+5}{3}} = x$

$-1 + \sqrt{\frac{y+5}{3}} = X$



range of $h(x)$ $[-2, +\infty)$

3. John bought his house in Bol in 2000 for \$290,000. House prices in Bol increase 7% ~~a year~~ every 2 years. Mary bought her house for \$ 400,000 in Flo in 2015. House prices in Flo double every 50 years . When will John house be worth twice as much as Mary 's ? Give the answers in years (Ex: in the year 2040)

$t=0$ corresponds to 2000

$f(t) = 290\sqrt{1.07}^{t}$ price of John's house in thousands

$g(t) = 400\left(\sqrt[50]{2}\right)^{t-15}$ price of Mary's house in thousands

want $f(t) = 2\,g(t)$

$$290\sqrt{1.07}^{t} = 2\cdot 400\sqrt[50]{2}^{\,t-15}$$

$$\left(\frac{\sqrt{1.07}}{\sqrt[50]{2}}\right)^{t} = \frac{800}{290}\frac{1}{\left(\sqrt[50]{2}\right)^{15}}$$

$$t = \frac{\ln\left(\frac{80}{29}\cdot\sqrt[50]{2}^{-15}\right)}{\ln\left(\frac{\sqrt{1.07}}{\sqrt[50]{2}}\right)} \approx 40$$

in 2040