

Lesson 12

Read Chapter 9

Inverse function

Given $f : A \rightarrow B$

the inverse function $f^{-1} : B \rightarrow A$ if it exists, is such that

$$f^{-1}(f(x)) = x, \quad f(f^{-1}(y)) = y$$

or $f(x) = y$ exactly when $f^{-1}(y) = x$

Example $f(x) = 2x$ Find f^{-1}

The graph of $f^{-1}(y)$ is the graph of $f(x)$ flipped around the line $y = x$

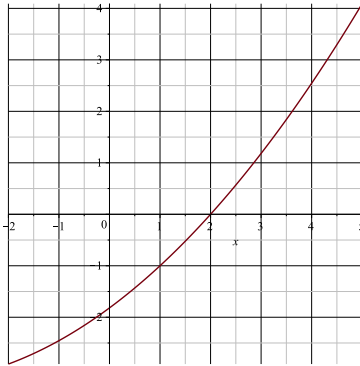
Domain $f^{-1} = \text{Range } f$
Range $f^{-1} = \text{Domain } f$

Does $f(x) = x^2$ have an inverse function ?

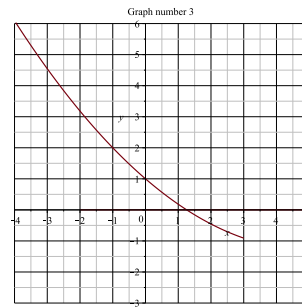
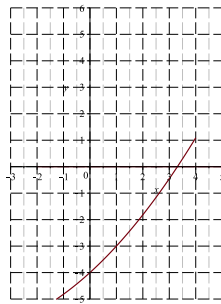
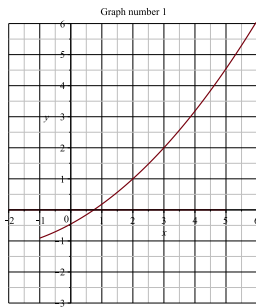
Let $f(x) = 2x - 1$ on the domain of all Real numbers. Is f invertible? If it is find the inverse, its domain and its range.

Given $g(x) = 2x - 1$ on the domain $0 \leq x \leq 5$. Is g invertible? If it is find the inverse, its domain and its range

2. Below is the graph of the function $y = f(x)$ on the domain $-2 \leq x \leq 5$



(a) Which of the graphs below is the graph of $y = 2 + f(x - 1)$? Circle the correct graph.



(b) If the domain of f is $-2 \leq x \leq 5$ what is the domain of the function $\frac{f(3x)+5}{x-1}$?

(c) Compute $f^{-1}(-1)$

(d) If $h(x) = e^{f(x)}$ Which of the values below is closest to $h^{-1}(2)$? Circle the the right answer.

0.6, -1, 2.5, -2, 3.5



Explain why $f(x) = -2x^2 + 60x$ is not invertible.

What is the inverse of $f(x) = -2x^2 + 60x$ on $[15, +\infty)$

What is the inverse of $f(x) = -2x^2 + 60x$ on $(-\infty, 15]$

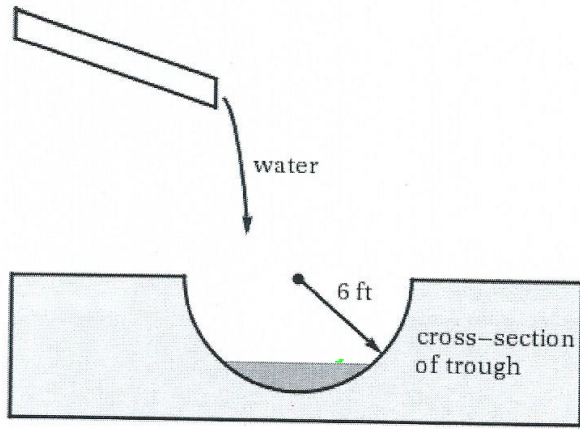
$\ln x$ is the inverse of e^x

$\arcsin x$ is the inverse of $\sin x$. Over which domain ?

Suppose p is the price of an item and $q = f(p)$ is the number of items sold at that price. Explain in words the meaning of:
 $f(25)$

$$f^{-1}(30)$$

A trough has a semicircular cross section with a radius of 6 feet. Water starts flowing into the trough in such a way that the depth of the water is increasing at a rate of 2 inches per hour.



(a) Give a function

$$w = f(t)$$

relating the width w , in feet of the surface of the water to the time t , in hours. Make sure to specify the domain and compute the range too.

(b) After how many hours will the surface of the water have width of 7 feet? (Round your answer to two decimal places.)

(c) Give a function

$$t = f^{-1}(w)$$

relating the time to the width of the surface of the water. Make sure to specify the domain and compute the range too.