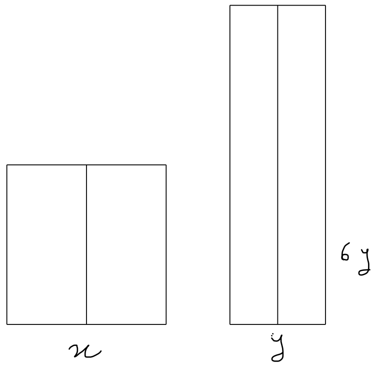
1. You want to build two enclosures using exactly 2000 meters of fencing. One enclosure will be a square, and the other will be a rectangle that is 6 times as long as it is wide. Each enclosure will be divided in half by a fence, as the picture below shows. What should the dimensions of the rectangular enclosure be in order to minimize the combined area of the two enclosures ? What dimensions maximize the combined area of the two enclosures?



want y, $6y$

$5x + 20y = 2000$

$x = \frac{2000-20y}{5} = 400-4y$

$x=0!$

$A = x^2 + 6y^2 = (400-4y)^2 + 6y^2 = 400^2 - 2\cdot 400\cdot 4y + 22y^2$

graph $\cup$ min at vertex $h = \frac{\cancel{2}\cdot 400\cdot \cancel{4}^{2}}{\cancel{2}\cdot \cancel{22}_{11}} = \frac{800}{11}$

$y = \frac{800}{11}$, $6y = \frac{4800}{11}$

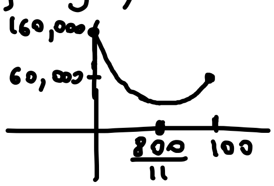
y is max when I build no square so when $20y = 2000$

What about max? $0 \le \underbrace{y \le 100}$ because y is a length

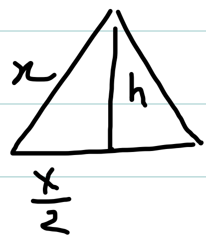
$A(0) = 160000$

$A(100) = 400^2 - 2\cdot 400\cdot 400 + 22\cdot 100^2 = 22\cdot 100^2 - 400^2 = 60,000$

Max for $y=0$, no rectangle



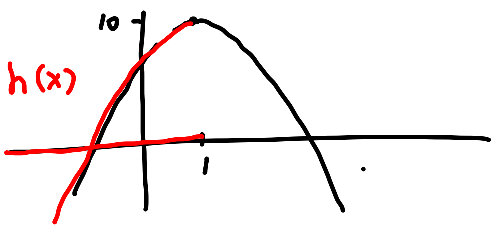
Area of equilateral triangle with side x



$$h^2 + \frac{x^2}{4} = x^2$$

$$h = \sqrt{\frac{3x^2}{4}} = \frac{x}{2}\sqrt{3}$$

$$A = \frac{1}{2}x \cdot \frac{1}{2}x \cdot \sqrt{3} = \frac{\sqrt{3}}{4}x^2$$



Problem 1 Let $f(x) = -(x-1)^2 + 10$ and $g(x) = \frac{x}{x+1}$.

a)(5 points) Compute $f(g(x))$ $= f\left(\frac{x}{x+1}\right) = -\left(\frac{x}{x+1} - 1\right)^2 + 10$

b)(5 points)Find an inverse for $\underset{h(x)}{f(x)}$ on the domain $x \le 1$

$y = -(x-1)^2 + 10$

$(x-1)^2 = 10 - y$

$(x-1) = \pm\sqrt{10-y}$

because $x \le 1$

$x = 1 - \sqrt{10-y}$

$h^{-1}(y) = 1 - \sqrt{10-y}$

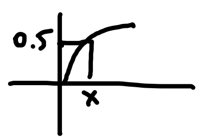
Domain of h^{-1} = Range of h = $(-\infty, 10]$

Range of h^{-1} = Domain of h $(-\infty, 1]$

c)(5 points)Suppose that $f(x)$, for $0 \le x \le 1$, gives you the altitude, at time x, of a ball that has been launched in the air. Time x is measured in seconds and altitude $f(x)$ in meters. Explain in words the meaning of $f^{-1}(0.5)$ (You do not need to compute the value of $f^{-1}(0.5)$)

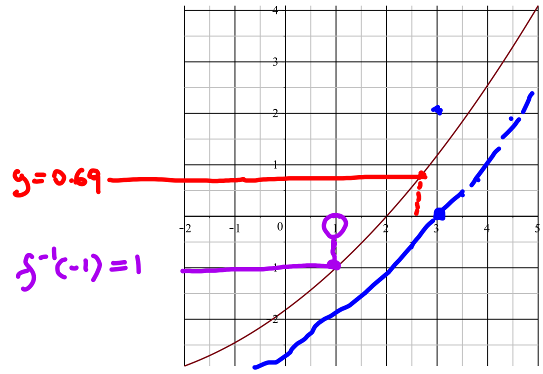
$h = f(x)$

$f^{-1}(0.5) = x$

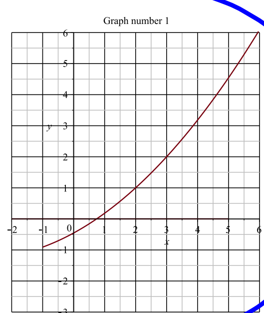


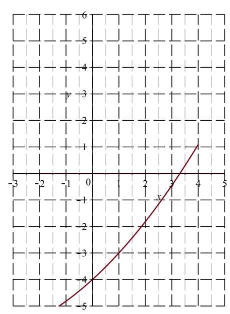
$f^{-1}(0.5)$ is the time for which the ball is at height 0.5 m

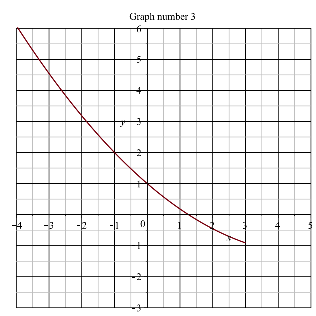
2. Below is the graph of the function $y = f(x)$ on the domain $-2 \le x \le 5$



(a) Which of the graphs below is the graph of $y = 2 + f(x-1)$? Circle the correct graph.







(b) If the domain of f is $-2 \le x \le 5$ what is the domain of the function $\frac{f(3x)+5}{x-1}$?

$x \neq 1$ $\quad -\frac{2}{3} \le x \le \frac{5}{3}$ $\quad$ (Since $f(3x)$ has domain $-2 \le 3x \le 5$, $-\frac{2}{3} \le x \le \frac{5}{3}$)

or $[-\frac{2}{3}, 1)$ and $(1, \frac{5}{3}]$

(c) Compute $f^{-1}(-1)$

$f^{-1}(-1) = 1$

(d) If $h(x) = e^{f(x)}$ Which of the values below is closest to $h^{-1}(2)$? Circle the the right answer.

0.6 ,-1, 2.5 -2, 3.5

$h(x) = e^{f(x)} = 2$

$x = h^{-1}(2)$

want x : $e^{f(x)} = 2$

$\ln(e^{f(x)}) = \ln(2)$

$f(x) = \ln(2) \approx 0.69$

so $x = f^{-1}(0.69)$

see graph of f

1. Mary has spiders and flies in her house. Yesterday she counted 5 spiders. The spider population increases 3% every week. Today Mary had 15 flies in her house; the flies population doubles every 5 days. When will there be 20 times more flies than spiders in Mary's house ?

$t=0$ today t measured in days

$s(t)=A_0 a^t$ spider population

$a = \sqrt[7]{1.03}$, through $(-1, 5)$ $\quad s(t) = 5\left(\sqrt[7]{1.03}\right)^{t+1}$

$= 5\sqrt[7]{1.03}\left(\sqrt[7]{1.03}\right)^t$

$f(t) = B_0 b^t$ fly population. $\quad b = \sqrt[5]{2} \quad B_0 = 15$

$f(t) = 15\left(\sqrt[5]{2}\right)^t$

$f(t) = 20 \cdot s(t)$

$15\left(\sqrt[5]{2}\right)^t = 20 \cdot 5\sqrt[7]{1.03}\left(\sqrt[7]{1.03}\right)^t$

$$\left(\frac{\sqrt[5]{2}}{\sqrt[7]{1.03}}\right)^t = \frac{20 \cdot 5\sqrt[7]{1.03}}{15}$$

$$t = \frac{\ln\left(\frac{20}{3}\sqrt[7]{1.03}\right)}{\ln\left(\frac{\sqrt[5]{2}}{\sqrt[7]{1.03}}\right)} \approx 14 \text{ days}$$

2. Write a formula for the function whose graph is the graph of $f(x) = 3(x+1)^2$ first shifted horizontally 5 units to the right, then scaled of a factor of $\frac{1}{2}$, then reflected around the x axis, then moved up 3 units.

$y = 3(x-5+1)^2$

$y = 3\left(\frac{x}{\frac{1}{2}} - 4\right)^2$

$y = -3(2x-4)^2$

$y = -3(2x-4)+3$

Write a formula for the function whose graph is the graph of $f(x) = 3(x+1)^2$ first scaled of a factor of $\frac{1}{2}$, then shifted horizontally 5 units to the right, then reflected around the x axis, then moved up 3 units.

$y = 3\left(\frac{x}{\frac{1}{2}} + 1\right)^2$

$y = 3(2(x-5)+1)^2$

$y = -3(2x-9)^2$

$y = -3(2x-9)^2 + 3$