Lesson 9

Read Chapter 7

Quadratic modelling

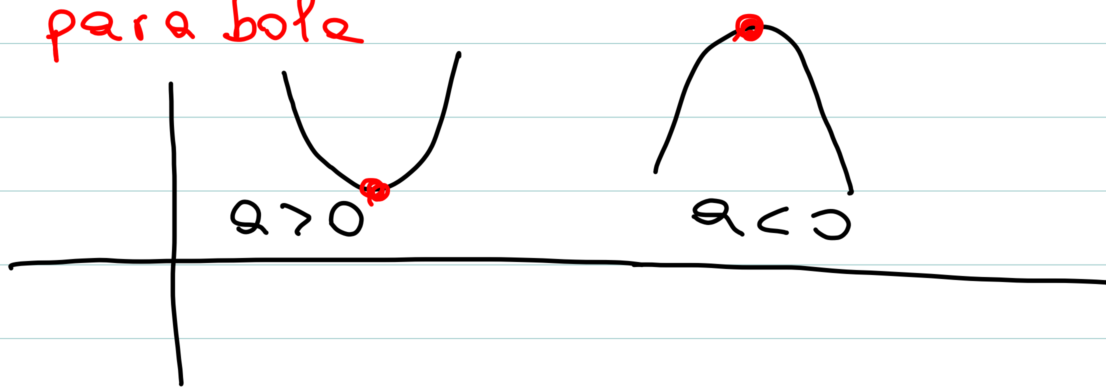
Recall

Quadratic function

$f(x) = ax^2 + bx + c$ or $f(x) = a(x-h)^2 + k$

$y =$

parabola

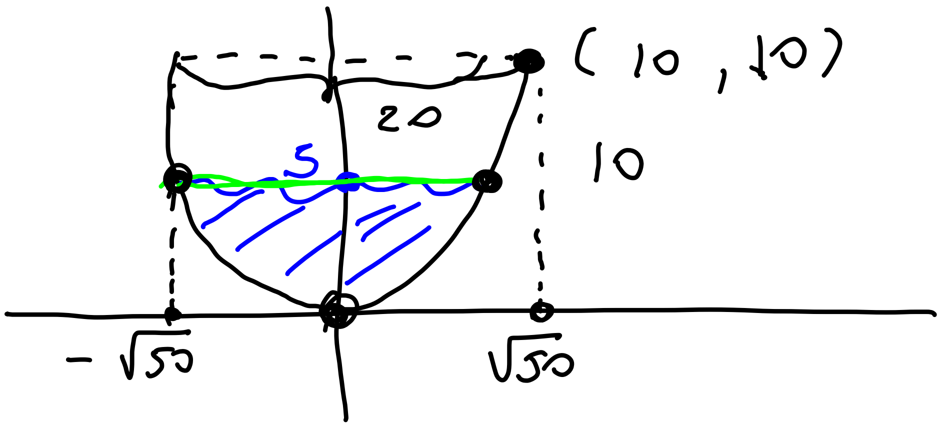


you need to know how to find the equation of a parabola

1) Through 3 points

2) Through 1 point, with given vertex

A drainage canal has a cross section in the shape of a parabola. Suppose the canal is 10 feet deep and 20 feet wide at its top. If the water depth in the ditch is 5 feet, how wide is the surface of the water in the ditch?



vertex $(\underset{h}{0}, \underset{k}{0})$, through $(10, 10)$

$y = ax^2$, $10 = a \cdot 10^2$, $\frac{10}{100} = a$, $\frac{1}{10} = a$

$y = \frac{1}{10}x^2$, plug in $y = 5$, $5 = \frac{1}{10}x^2$, $50 = x^2$

$\pm\sqrt{50} = x$; $\boxed{\text{width} = 2\sqrt{50}}$

Enrollment is an online course is modeled by a quadratic function. At the beginning of the quarter ($t = 0$) 300 students are enrolled in the class. Five days later ($t = 5$) 450 students are enrolled. Twenty five days later ($t = 25$) only 50 students are enrolled. The class is terminated when it has no more students. When is it terminated ? What was the maximum number of students enrolled?

$(0, 300) \quad (5, 450), \quad (25, 50)$

$y = f(t)$ $\quad$ y = # students, $\quad t$ = # days after quarter started

$y = at^2 + bt + c$

$$
\begin{aligned}
300 &= c \\
450 &= a \cdot 5^2 + b \cdot 5 + c \\
50 &= a \cdot 25^2 + b \cdot 25 + c
\end{aligned}
$$

$$\begin{cases} 300 = c \\ 450 = a \cdot 25 + b \cdot 5 + c \\ 50 = a \cdot 625 + b \cdot 25 + c \end{cases}$$

$$\begin{cases} c = 300 \\ 450 = 25 \cdot a + 5 \cdot b + 300 \\ 50 = 625 \cdot a + 25 \cdot b + 300 \end{cases}$$

$$\begin{cases} c = 300 \\ \dfrac{150 - 25a}{5} = b \\ 50 = a \cdot 625 + \overbrace{(30 - 5a)}^{b} \cdot 25 + 300 \end{cases}$$

$$30 - 5a = b$$

$$\begin{cases} c = 300 \\ b = 30 - 5a \\ 50 = 625a - 125a + 750 + 300 \end{cases}$$

$$\rightarrow -1000 = 500a \rightarrow a = -2$$

$$c = 300$$

$$c = 300$$
$$b = 30 - 5(-2) = 40$$
$$a = -2$$

students

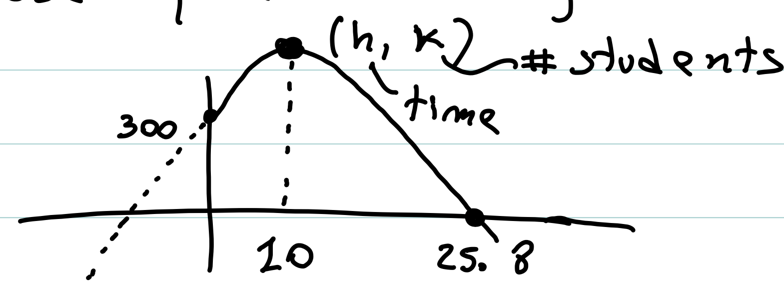
$$y = -2t^2 + 40t + 300$$

days after beginning of quarter

No students when:
set $y=0$, solve for t

$$0 = -2t^2 + 40t + 300$$

Use quadratic formula : $t = 25.8$, $\cancel{-5.8}$



Max number of students:

$$h = -\frac{b}{2a} = -\frac{40}{2\cdot(-2)} = 10$$

$$k = -2(10)^2 + 40\cdot 10 + 300 = \boxed{500}$$

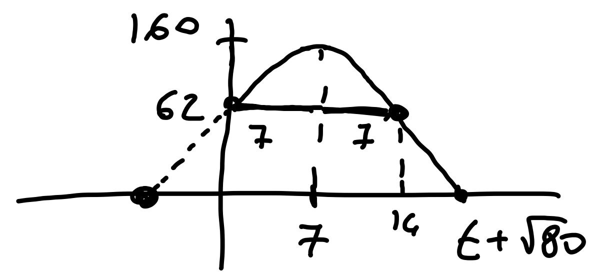
Enrollment in an online course is modeled by a quadratic function. At the beginning of the quarter ($t = 0$) 62 students are enrolled in the class. one week later ($t = 7$) the class reaches its maximum enrollment of 160 students. How many students are there in the class at $t = 14$? When is the class terminated?

$(0, 62)$ $\quad (\underset{h}{\overset{\shortparallel}{7}}, \underset{k}{\overset{\shortparallel}{160}})$ = vertex

$y = f(t)$ $\quad y$ = # students , t = # days after quarter started

$y = a(t-7)^2 + 160$; $\quad 62 = a(0-7)^2 + 160$; $\quad -98 = 49a$

$a = -\frac{98}{49} = -2$



$y = -2(t-7)^2 + 160$

① $y = -2(14-7)^2 + 160 = 62$

② $0 = -2(t-7)^2 + 160$

$\frac{2}{2}(t-7)^2 = \frac{160}{2}$; $\quad t - 7 = \pm\sqrt{80}$ $\quad t = 7 + \sqrt{80}$

≈ 15.94

Math 120 (Pezzoli)
Fall 2019
Midterm #1

Name ________________

TA: ________________

Section: ________________

Instructions:

- Your exam contains 3 problems.
- Your exam should contain 4 pages; please make sure you have a complete exam.
- Box in your final answer when appropriate.
- Unless stated otherwise, you **MUST** show work for credit. No credit for answers only. If in doubt, ask for clarification.
- Your work needs to be neat and legible.
- You are allowed one 8.5×11 sheet of notes (both sides).
- The only calculator allowed is the Ti-30x IIS.
- Round off your answers to ~~3~~ 2 decimal places, unless you are asked for exact answers.

Problem #1 [redacted] ________

Problem #2 [redacted]) ________

Problem #3 [redacted]) ________

TOTAL [redacted]) __________

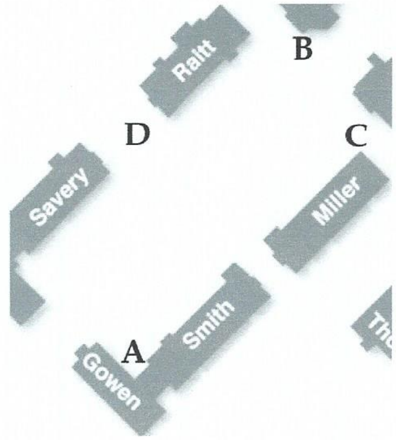
Week 3 worksheet

8. At noon, Alex exits Smith Hall at point A on the map shown and starts walking at constant speed directly towards the Art building (point B), hoping for a cup of coffee at Parnassus. She gets to point B after 60 seconds.

At the same time (noon), Matt is at point C (near the Music building), walking straight towards point D at a uniform speed of 2 feet per second, rushing to his next class.

Point B is 60 feet east and 110 feet north of point A. Point D is 80 feet due north of point A, and point C is 70 feet due east of point D.

Impose a coordinate system with the origin at point A.



(a) Determine parametric equations for Alex's coordinates t seconds past noon.

$$\left(t, \frac{11}{6} t\right)$$

(b) Determine parametric equations for Matt's coordinates t seconds past noon.

$$(70 - 2t, 80)$$

.. Find minimum distance .

~~(c)~~ What is the closest distance between Matt and Alex during their treks across the Quad?

SKIP (NEXT WEEK)

$$d(\text{Alex}, \text{Matt}) = \sqrt{(70 - 2t - t)^2 + \left(80 - \frac{11t}{6}\right)^2}$$

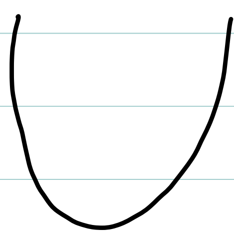
$$(70 - 3t)^2$$

$$d = \sqrt{70^2 - 2 \cdot 70 \cdot 3 \cdot t + 9t^2 + 80^2 - 2 \cdot 80 \cdot \frac{11}{6} t + \frac{121}{36} t^2}$$

$$d(t) = \sqrt{\frac{445}{36} \cdot t^2 - \frac{2140}{3} \cdot t + 11300}$$

$$d^2(t) = \frac{445}{36} t^2 - \frac{2140}{3} t + 11300$$

Trick: the value of t that minimizes/maximizes $d^2(t)$ also minimizes/maximizes $d(t)$



$d^2(t)$ is quadratic

so minimum when $t = \dfrac{\frac{2140}{3}}{2 \cdot \frac{445}{36}}$

$$= \frac{2140}{3} \cdot \frac{36}{2 \cdot 445} \approx 28.85393258$$

now calculate $d(28.8539325)$

make sure you plug in in $d(t)$ not $d^2(t)$; $d(28.85393258) =$

$$\sqrt{445/36\,(28.85393258)^2 - \frac{2140}{3} \cdot 28.85393258 + 11300}$$

$$\approx \boxed{31.7610 \text{ feet}}$$