Lesson 8

Read Chapter 7

Quadratic functions. Parabolas

A quadratic function is a function given by a quadratic formula :

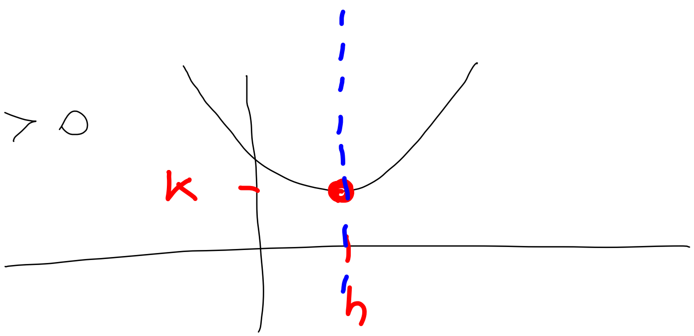
$$f(x) = ax^2 + bx + c \quad a \neq 0$$

standard formula

$$y = ax^2 + bx + c$$

The graph of a quadratic function is a parabola

$a > 0$



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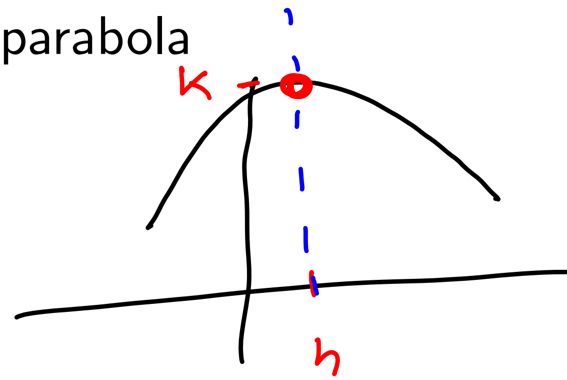


$a < 0$

vertex (h, k)

$x = h$ is axis of symmetry

vertex

The vertex of a parabola is a point (h, k) that is either the highest (when $a < 0$) or the lowest (when $a > 0$) point of the parabola

Vertex form: $y = a(x - h)^2 + k$

$x = h$ is the axis of symmetry for a parabola with vertex (h, k)

The parabola

$$f(x) = ax^2 + bx + c$$

has vertex

$$h = -\frac{b}{2a} \quad k = f(-\frac{b}{2a})$$

From vertex form to standard form

$$y = a(x-h)^2 + k$$

$$y = a\left(x^2 - 2x \cdot h + h^2\right) + k$$

$$y = \underbrace{a}_{a} x^2 \underbrace{-2ah}_{b} x + \underbrace{ah^2 + k}_{c}$$

From standard form to vertex form

$$y = \underline{\underline{a}}x^2 + bx + c$$

$$y = \underline{\underline{a}}(x-h)^2 + k \qquad h = -\frac{b}{2a} \qquad k = a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + c$$

Given the parabola $y = \underbrace{2}_{a}x^2 + \underbrace{1}_{b} \cdot x + \underbrace{6}_{c}$, put it in vertex form and draw it.

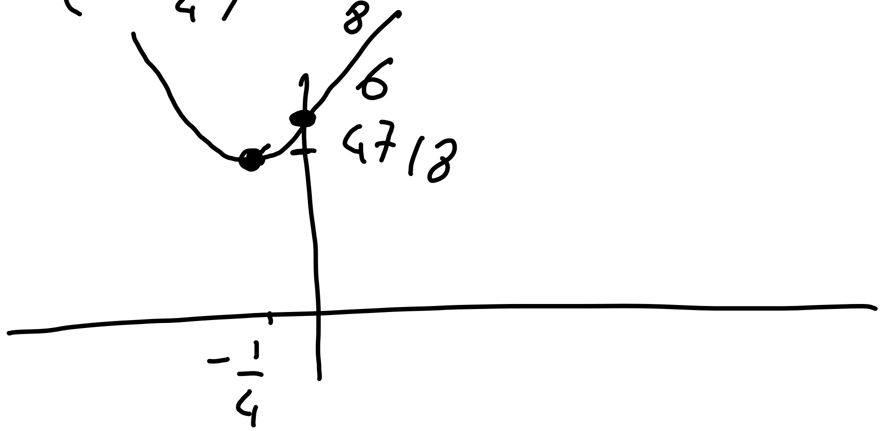
$$y = a(x-h)^2 + k$$

$$a = 2$$

$$h = -\frac{b}{2a} = \frac{-1}{2 \cdot 2} = -\frac{1}{4}$$

$$k = 2\left(-\frac{1}{4}\right)^2 + \left(-\frac{1}{4}\right) + 6 = \frac{47}{8}$$

$$y = 2\left(x + \frac{1}{4}\right)^2 + \frac{47}{8}$$



Parabola through three points

Find the equation of the parabola through (1,2),(-1,1) and (2,3)

$y = ax^2 + bx + c$ Find a, b, c

$$\begin{cases} 2 = a \cdot 1^2 + b \cdot 1 + c \\ 1 = a(-1)^2 + b(-1) + c \\ 3 = a \cdot 2^2 + b \cdot 2 + c \end{cases}$$

solve this system

$$\begin{cases} 2 = a + b + c \\ 1 = a - b + c \\ 3 = 4a + 2b + c \end{cases} \qquad \begin{cases} a = 2 - b - c \\ 1 = (2 - b - c) - b + c \\ 3 = 4(2 - b - c) + 2b + c \end{cases}$$

$$\begin{cases} a = 2 - b - c \\ 1 = 2 - 2b \\ 3 = 8 - 2b - 3c \end{cases} \qquad \begin{cases} b = \frac{1}{2} \\ 3 - 8 + 2\,\frac{1}{2} = -3c \qquad \frac{-4}{-3} = c \\ a = 2 - \frac{1}{2} - \frac{4}{3} = \frac{1}{6} \end{cases}$$

$$y = \frac{1}{6} x^2 + \frac{1}{2} x + \frac{4}{3}$$

Find the equation of the parabola with vertex (1,2) through the point (4,5)

h k

$$y = a(x-h)^2 + K$$

$$y = a(x-1)^2 + 2$$

$$5 = a(4-1)^2 + 2$$ solve for a

$$5-2 = a \cdot 9$$

$$\frac{3}{9} = a$$

$$\frac{1}{3} = a$$

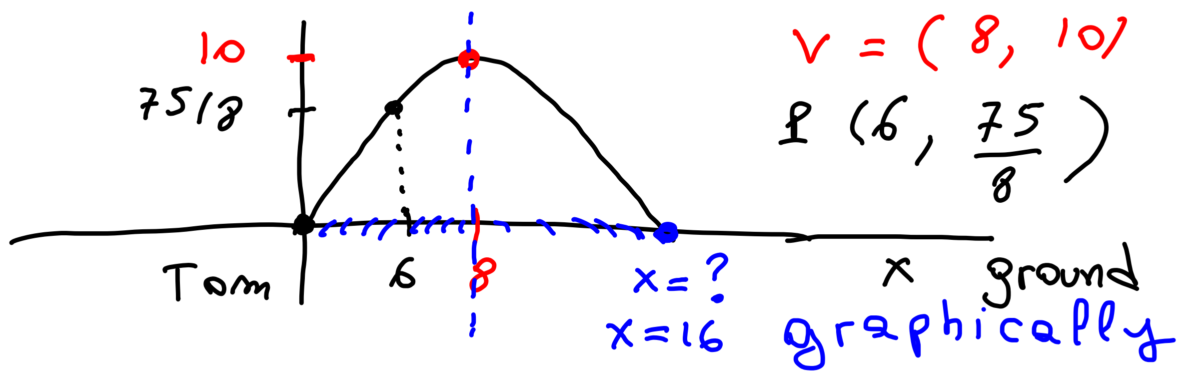
$$y = \frac{1}{3}(x-1)^2 + 2$$

Fact: the minimum value of $f(x) = ax^2 + bx + c \quad (a > 0)$ is at the vertex

so we have a min at $x = -\frac{b}{2a}$

min value $y = f(-\frac{b}{2a})$

Tom kicks a ball. When the ball is 6 feet to the east of Tom the ball height is $\frac{75}{8}$ feet, the ball reaches a maximum height of 10 feet 8 feet to the East of Tom. How far to the East of Tom does the ball fall back to the ground ? The ball's trajectory is a parabola.



$V = (8, 10)$

$P\left(6, \frac{75}{8}\right)$

Using algebra: plan: ① find equation of parabola

② Find x intercepts for parabola

$y = a(x-8)^2 + 10$

$\frac{75}{8} = a(6-8)^2 + 10$; $\frac{\frac{75}{8} - 10}{4} = \frac{a \cdot 4}{4}$ $\quad a = -\frac{5}{32}$

$$y = -\frac{5}{32}(x-8)^2 + 10$$

To find x intercepts: $0 = -\frac{5}{32}(x-8)^2 + 10$

$$\frac{\cancel{32}}{\cancel{5}}\cdot\frac{\cancel{5}}{\cancel{32}}(x-8)^2 = 10\cdot\frac{32}{5}$$

$$x-8 = \pm\sqrt{\frac{320}{5}} = \pm\sqrt{64}$$

$$x = 8 \pm 8 \qquad x = 0,\ 16$$

$$f(x) = \begin{cases} x & \text{if } x < 2 \\ \sqrt{9-(x-2)^2} - 5 & \text{if } 2 \le x \le 5 \end{cases}$$

$$-5 + \sqrt{9-(x-2)^2}$$

$C(2,-5)$ $r = 3$

$y = x$