Lesson 6

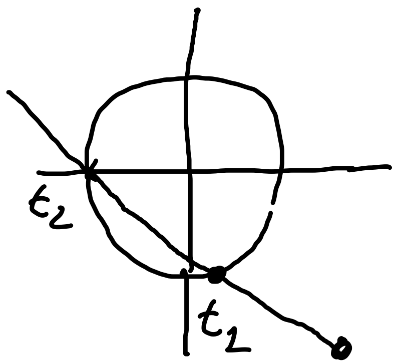
Read Chapter 5

Functions,domain range, inverse

A crop dusting airplane flying a constant speed of 120mph is first spotted 2 miles South and 1.5 miles East of the center of circular irrigated field. The irrigated field has radius 1 mile. The plane flies in a straight line to a point 1 mile West of the center of the irrigated field.

Find the location A where the crop duster enters the airspace above the field

$$y = -2 - 0.8(x - 1.5)$$



$P\ (1.5, -2)$

$t = 0$

When does the plane first enter the airspace above the field ?
(Assume time t=0 corresponds to when the plane is first spotted)

Parametric equations of plane:

$$\left.\begin{aligned} x &= 1.5 - 93.6330\,t \\ y &= -2 + 74.9064\,t \end{aligned}\right\}$$ from Past time

Field $\quad x^2 + y^2 = 1$

$(1.5 - 93.6330\,t)^2 + (-2 + 74.9064\,t)^2 = 1 \qquad$ solve for t

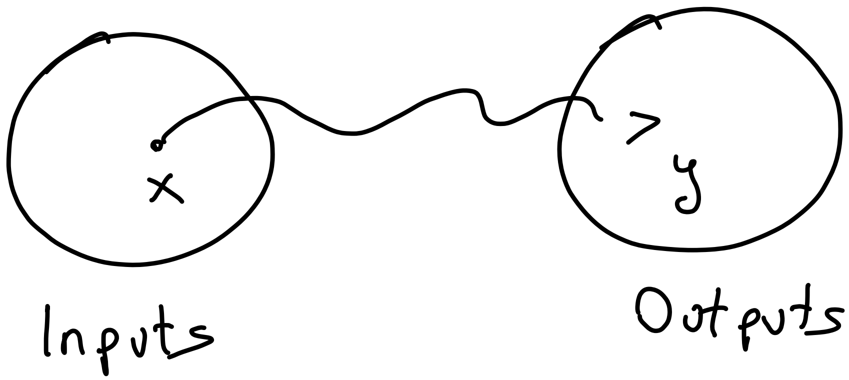
$14\,378.1074\,t^2 - 580.5246\,t + 6.25 = 0$

$t = 0.0267\,,\ \boxed{0.0137}$

How much time does the plane spend flying over the irrigated field?

$$\underset{t_2}{0.0267} - \underset{t_1}{0.0137} = 0.013 \text{ hr}$$

What is a function?



Domain and Range

Domain : all inputs

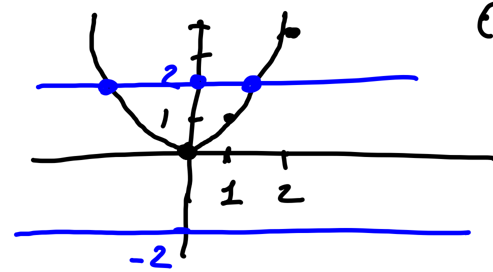
Range : all outputs

Example: $f(x) = x^2$

Domain : all number x for which $f(x)$ makes sense

In this example . domain is "all x"

Range: look at graph



$(x, y = f(x))$
x in domain

all y values k
s.t the line $y = k$
intersects graph of function

In this example range is $y \geq 0$

Interval notation

$(2, 3)$ means all x with $2 < x < 3$
$[2, 3]$ means all x with $2 \leq x \leq 3$
$[2, 3)$ means all x with $2 \leq x < 3$
$(-\infty, +\infty)$ means all x in R
$(-\infty, 1]$ $x \leq 1$

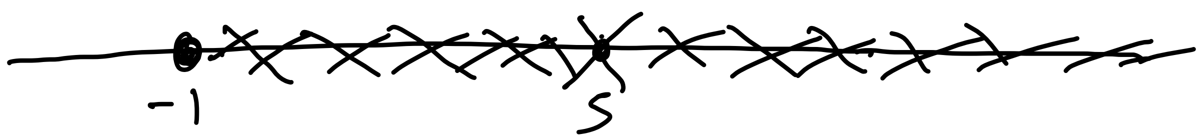
Find the (natural) domain of $f(x) = \frac{\sqrt{x+1}}{x-5}$

$x \neq 5$

$x+1 \geq 0$

$x \geq -1$

$(\sqrt{0} = 0)$



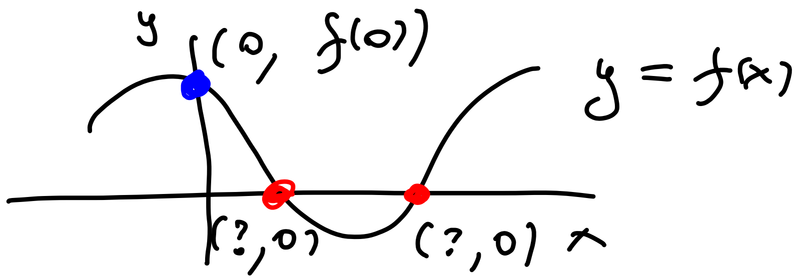
$[-1, 5)$ and $(5, +\infty)$

$\ln\left(\text{EXPR}\right)$ requires EXPR > 0

$\sqrt{\text{EXPR}}$ requires EXPR ≥ 0

$\frac{\text{SOMETHING}}{\text{EXPR}}$ requires EXPR $\neq 0$

x and y intercepts



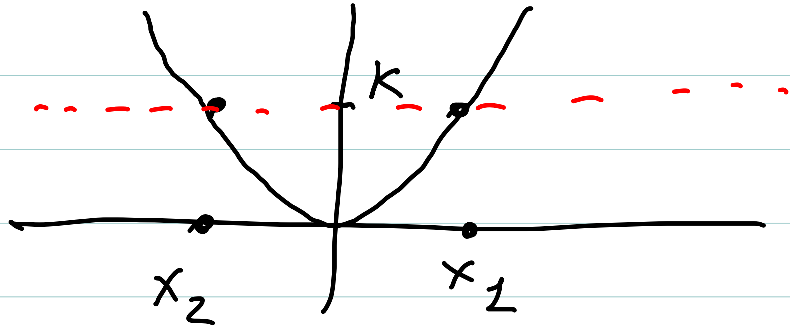
Given $y = f(x)$
To find y intercept calculate $f(0)$
To find x intercept(s) set $f(x) = 0$ and solve for x

Find x and y intercepts for $\overset{y =}{f(x)} = x^2 - 5x + 6$

y intercept: $x=0$ $f(0)=6$ $(\underline{\underline{0}}, 6)$

x intercepts: $y=0$ $0 = x^2 - 5x + 6$ solve for x

$x = 2, 3$ $(2, \underline{\underline{0}})$ $(3, \underline{\underline{0}})$

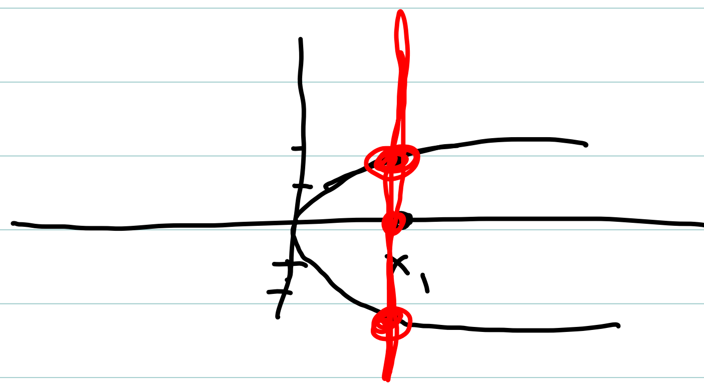


Does not satisfy horizontal line test

$f(x) = x^2$

$f(2) = 4$ ok
$f(-2) = 4$

1–1



$f(4) = 2$
$f(4) = -2$

Not graph of function

Algebraic manipulations

find an expression for

Given $f(x) = \frac{\sqrt{x+1}}{x-5}$ ~~calculate~~ $f(1+h)$

$$f(1+h) = \frac{\sqrt{(1+h)+1}}{(1+h)-5} = \frac{\sqrt{h+2}}{h-4}$$

Given $f(x) = \sqrt{x-8}$ simplify $\frac{f(x+h)-f(x)}{h}$ enough so that plugging in $h=0$ is allowed

$$f(z) = \sqrt{z - 8}$$

$$\frac{\overbrace{\sqrt{(x+h)-8}}^{a} - \overbrace{\sqrt{x-8}}^{b}}{h} \cdot \frac{\sqrt{(x+h)-8} + \sqrt{x-8}}{\sqrt{(x+h)-8} + \sqrt{x-8}} = \frac{\overbrace{(\cancel{x}+h) \cancel{-8}}^{a^2} - \overbrace{(\cancel{x}\cancel{-8})}^{b^2}}{h\left(\sqrt{(x+h)-8} + \sqrt{x-8}\right)}$$

$$\frac{\cancel{h}\ 1}{\cancel{h}\left(\sqrt{(x+h)-8} + \sqrt{x-8}\right)} = \frac{1}{\sqrt{(x+h)-8} + \sqrt{x-8}}$$

if $h = 0$

$$\frac{1}{\sqrt{x-8} + \sqrt{x-8}} = \frac{1}{2\sqrt{x-8}}$$

Recall $(a-b)(a+b) = a^2 - b^2$