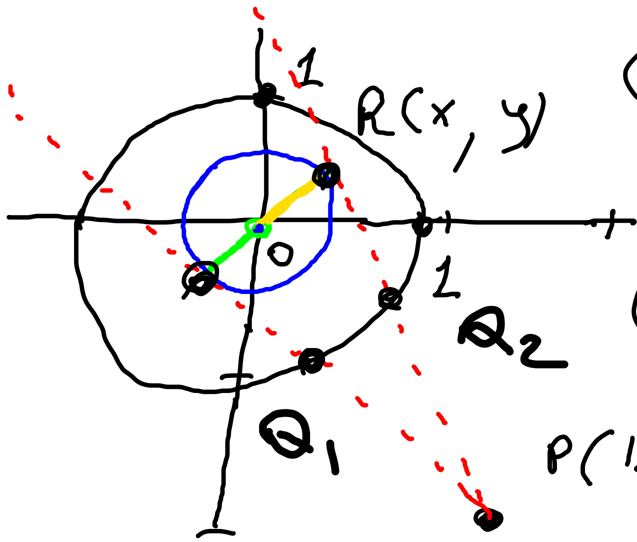
Lesson 5

Read Chapter 4

Parametric coordinates of motion for linear motion

Where should the plane enter the field if we want it to get 0.5 miles to the center (still flying in a straight path), but no closer?



① Find tangent lines to circle of radius $\frac{1}{2}$

② Find intersection between tangent lines $P(1.5, -2)$ and $x^2 + y^2 = 1$

slope OR $\dfrac{y-0}{x-0} = \dfrac{y}{x}$

Slope tangent $- \dfrac{1}{\frac{y}{x}} = -\dfrac{x}{y}$

$$\begin{cases} x^2+y^2=\frac{1}{4} \\ -\frac{x}{y}=\frac{y+2}{x-1.5} \end{cases} \qquad \begin{cases} x^2+y^2=1/4 \\ -x^2+1.5x=y^2+2y \end{cases}$$

$$\begin{cases} x^2+y^2=1/4 \\ 1.5x-2y=x^2+y^2 \end{cases}$$

$$\begin{cases} x^2+y^2=1/4 \\ 1.5x-2y=\frac{1}{4} \end{cases}$$

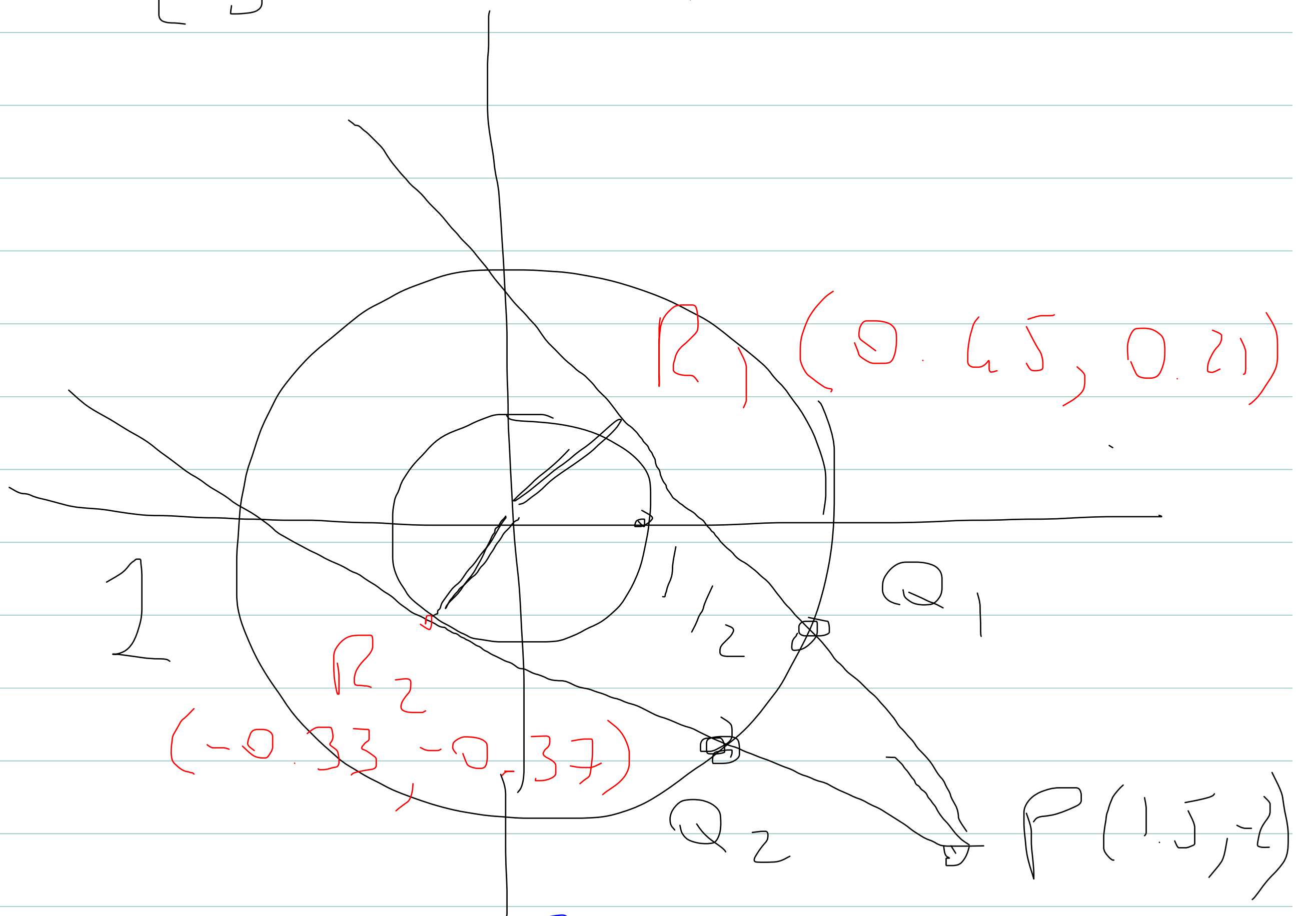
$$\begin{cases} x^2+y^2=1/4 \\ \frac{1.5}{2}x-1/8=y \end{cases}$$

$$\frac{1.5}{2} = 3/4$$

$$\begin{cases} x^2+\left(3/4\,x-\frac{1}{8}\right)^2=1/4 \\ 3/4\,x-\frac{1}{8}=y \end{cases}$$

$$\begin{cases} y = \frac{3}{4}x - \frac{1}{8} \\ 25x^2 - 3x - 15 = 0 \end{cases}$$

$$\begin{cases} x = 0.45, \quad -0.33 \\ y = 0.21, \quad -0.37 \end{cases}$$



$C : x^2 + y^2 = 1$

First tangent line y_1 is line ℓ_1 through $P\,(1.5, -2)$ and $R_1\,(0.45, 0.21)$

$$y = 0.21 + \frac{-2 - 0.21}{1.5 - 0.45}(x - 0.45)$$

$$y = 0.21 - 2.1(x - 0.45)$$

Q_1 is intersection of ℓ_1 and circle $x^2 + y^2 = 1$

$$\begin{cases} y = 0.21 - 2.1(x - 0.45) \\ x^2 + y^2 = 1 \end{cases}$$

$$\begin{cases} y = 0.21 - 2.1(x - 0.45) \\ x^2 + \left(0.21 - 2.1(x - 0.45)\right)^2 = 1 \end{cases}$$

$$\begin{cases} y = 0.21 - 2.1\,(x - 0.45)^2 \\ 5.41x^2 - 4.85x + 0.33 = 0 \end{cases}$$

$$\begin{cases} x = 0.82\ ,\ 0.07 \\ y = -0.567\ ,\ 1.008 \end{cases}$$

What solution do I want? Look at picture

$Q_1 = (0.82, -0.567)$

Plane can enter at Q_1

We can also do similar calculations to find Q_2 (I will skip it) and plane could enter at Q_2

The parametric equation of motion of a moving object are a pair of equations of the form

$$x(t) = \text{formula in } t$$

$$y(t) = \text{formula in } t$$

They give us the coordinates of the object at time t

Parametric equations. Uniform rectilinear motion.

Suppose an object is at $P(x_1, y_1)$ at time t_1 and it moves along a straight line at constant speed v.
The parametric equations of motion of the object are :

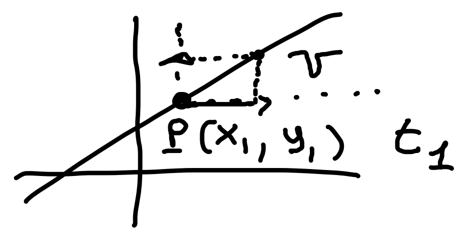
$$x = x_1 + v_x(t - t_1), \qquad y = y_1 + v_y(t - t_1)$$

where v_x is the horizontal component of the velocity and v_y is the vertical component of the velocity.

v

velocity

speed.

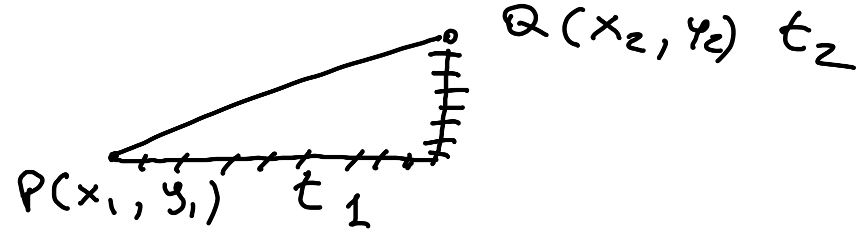


You can calculate v_x and v_y in different ways, depending on what the problem gives you :

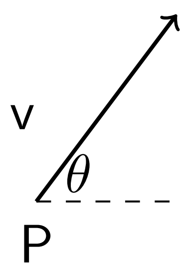
- If you also know the object is at $Q(x_2, y_2)$ at time t_2 then

  $v_x = \frac{x_2 - x_1}{t_2 - t_1} \left(\frac{\Delta x}{\Delta t}\right)$

  $v_y = \frac{y_2 - y_1}{t_2 - t_1} \left(\frac{\Delta y}{\Delta t}\right)$



- If you know v and θ (see figure) then

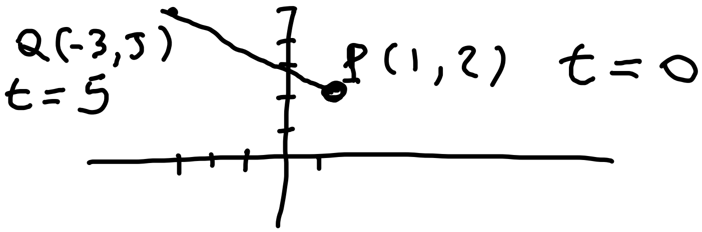


$$v_x = v\cos(\theta)$$
$$v_y = v\sin(\theta).$$

Note: in many problems time t_1 is just the initial time so $t_1 = 0$ in which case you have

$$x = x_1 + v_x t, \qquad y = y_1 + v_y t$$

Alice is running in the xy plane. She runs in a straight line from the point (1,2) to the point (-3,5) taking 5 seconds. Find her equations of motion.

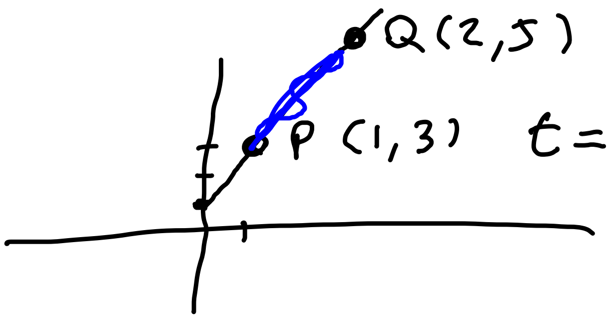


$$x = 1 + v_x(t-0)$$
$$y = 2 + v_y(t-0)$$

$$v_x = \frac{-3-1}{5-0} = -\frac{4}{5}$$
$$v_y = \frac{5-2}{5-0} = \frac{3}{5}$$

$$x = 1 - \frac{4}{5}t$$
$$y = 2 + \frac{3}{5}t$$

Alice is running at a speed of 5mi/hr starting at $P(1,3)$ along the line $y = 2x + 1$ in the NE direction. What are Alice' s parametric equations of motion ?



$t = \frac{\sqrt{5}}{5}$

$t = 0$

Choose Q

$x = 2 \quad , \quad y = 2 \cdot 2 + 1 = 5$

$Q(2, 5)$

$$d = v \cdot t$$

$$t = \frac{d}{v} = \frac{d(PQ)}{5} = \frac{\sqrt{(2-1)^2 + (5-3)^2}}{5} = \frac{\sqrt{5}}{5}$$

$$x = 1 + v_x t$$
$$y = 3 + v_y t$$

$$v_x = \frac{2-1}{\frac{\sqrt{5}}{5} - 0} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

$$v_y = \frac{5-3}{\frac{\sqrt{5}}{5} - 0} = 2 \cdot \frac{5}{\sqrt{5}} = 2\sqrt{5}$$

When is Alice 's 4 mi away from the point $Q(2,2)$?

$$x = 1 + \sqrt{5}\,t$$
$$y = 3 + 2\sqrt{5}\,t \qquad (1+\sqrt{5}\,t,\ 3+2\sqrt{5}\,t)$$

$$d\big((1+\sqrt{5}\,t,\ 3+2\sqrt{5}\,t),\ (2,2)\big) = 4$$

$$\sqrt{(1+\sqrt{5}\,t-2)^2 + (3+2\sqrt{5}\,t-2)^2} = 4$$

$$(-1+\sqrt{5}\,t)^2 + (1+2\sqrt{5}\,t)^2 = 16$$

$$1 + 2(-1)\sqrt{5}\,t + 5t^2 + 1 + 2\cdot 1\cdot 2\sqrt{5}\,t + 20t^2 = 16$$

$$25t^2 + 2\sqrt{5}\,t - 14 = 0$$

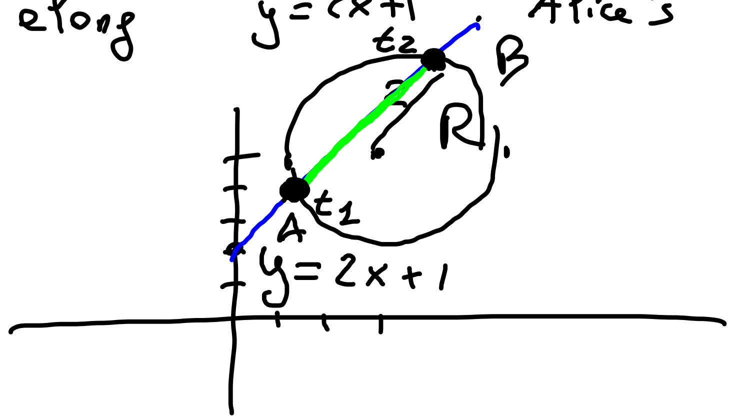
use quadratic formule

$$t = -0.84, \quad \boxed{t = 0.66} \text{ hrs.}$$

A construction site is located at $R(3,5)$ and construction noise can be heard up to 2 mi away from the site. For how long does Alice hear~~s~~ construction noise ?

time

Recall $A(t) = (1+\sqrt{5}t,\ 3+2\sqrt{5}t)$, Alice moves along $y = 2x+1$ Alice's speed is 5 mph



$$(x-3)^2 + (y-5)^2 = 2^2$$

$$(1+\sqrt{5}t - 3)^2 + (3 + 2\sqrt{5}t - 5)^2 = 2^2$$

solve for t $\quad t_1 = 0.1789 \quad t_2 = 0.8944$

$t_2 - t_1 = 0.72$ hrs

Method 2

1) Find A and B

Solve
$$\begin{cases} y = 2x+1 \\ (x-3)^2 + (y-5)^2 = 4 \end{cases}$$

$$\begin{cases} y = 2x+1 \\ (x-3)^2 + (\underbrace{2x+1-5}_{(2x-4)^2})^2 = 4 \end{cases}$$

$$\begin{cases} x^2 - 6x + 9 + 4x^2 - 16x + 16 = 4 \\ y = 2x+1 \end{cases}$$

$$\begin{cases} 5x^2 - 22x + 21 = 0 \\ y = 2x + 1 \end{cases}$$

$$x = \frac{22 \pm \sqrt{(22)^2 - 4 \cdot 5 \cdot 21}}{10} \begin{cases} 3 \\ 7/5 \end{cases} \qquad y = \begin{cases} 7 \\ 19/5 \end{cases}$$

$A(3, 7)$ $B(7/5, \frac{19}{5})$. Find distance AB

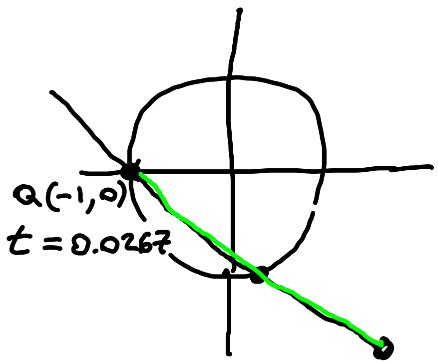
$$\sqrt{\left(3 - \frac{7}{5}\right)^2 + \left(7 - \frac{19}{5}\right)^2} \simeq 3.5777$$

t to travel that distance is $t = \frac{3.5777}{5} \approx 0.72$

speed $\to$ 5

A crop dusting airplane flying a constant speed of 120mph is first spotted 2 miles South and 1.5 miles East of the center of circular irrigated field. The irrigated field has radius 1 mile. The plane flies in a straight line to a point 1 mile West of the center of the irrigated field.

Find the location A where the crop duster enters the airspace above the field



$P\ (1.5, -2)$

$t = 0$

$y = -2 - 0.8(x - 1.5)$

$t = \dfrac{d(PQ)}{120} = \dfrac{\sqrt{(-1-1.5)^2 + 2^2}}{120} = 0.0267$

$x = 1.5 + v_x t$

$y = -2 + v_y t$

$v_x = \dfrac{-1 - 1.5}{0.0267 - 0} = -93.6330$

$v_y = \dfrac{0 - (-2)}{0.0267 - 0} = 74.9064$

When does the plane first enter the airspace above the field ?
(Assume time t=0 corresponds to when the plane is first spotted)

$$x = 1.5 - 93.6330t$$

$$y = -2 + 74.9064t$$

Field : $x^2 + y^2 = 1$

$$(1.5 - 93.6330t)^2 + (-2 + 74.9064t)^2 = 1$$

We will finish next time