Lesson 21

Read Chapter 18

trigonometric functions

and inverse trigonometric functions

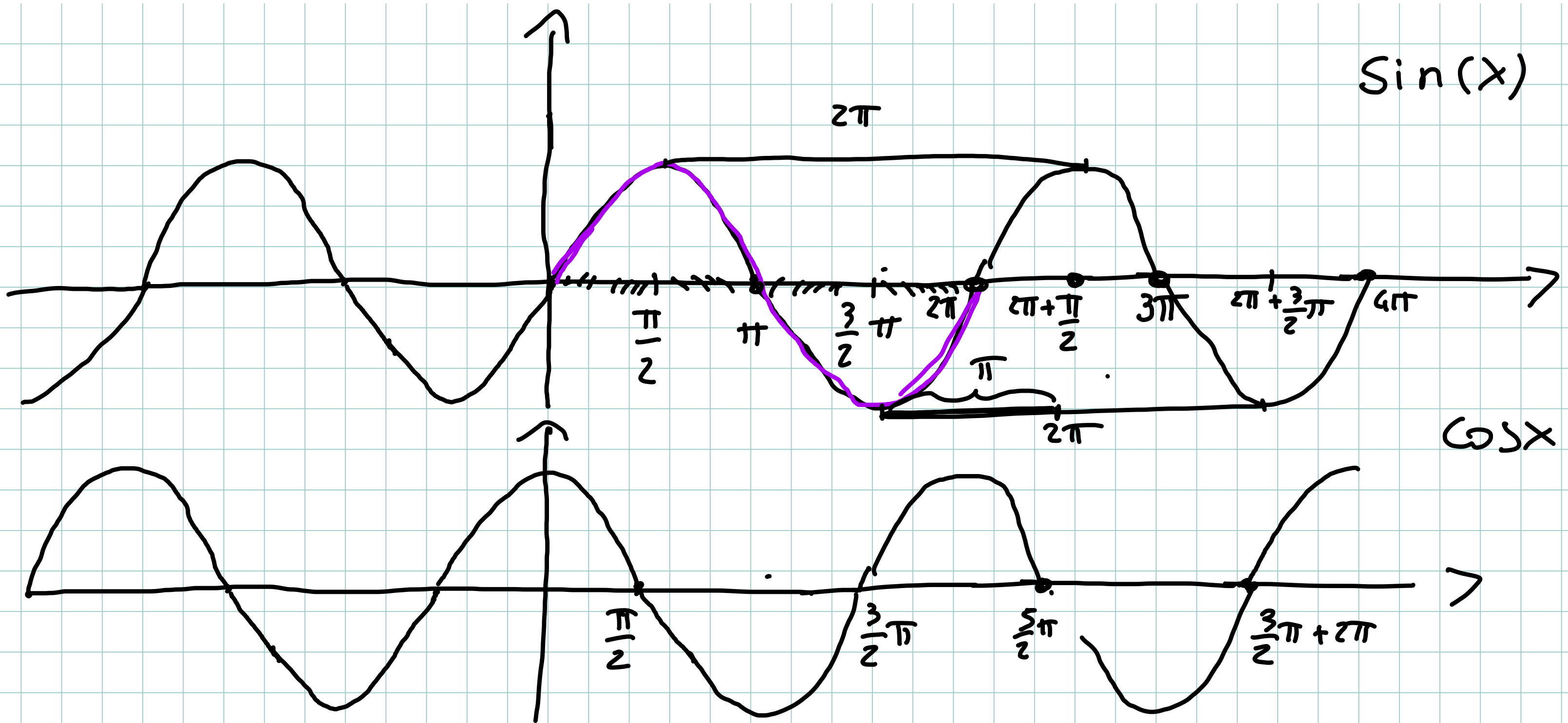
$$\tan x = \frac{\sin x}{\cos x}$$

$$\operatorname{cotan}(x) = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

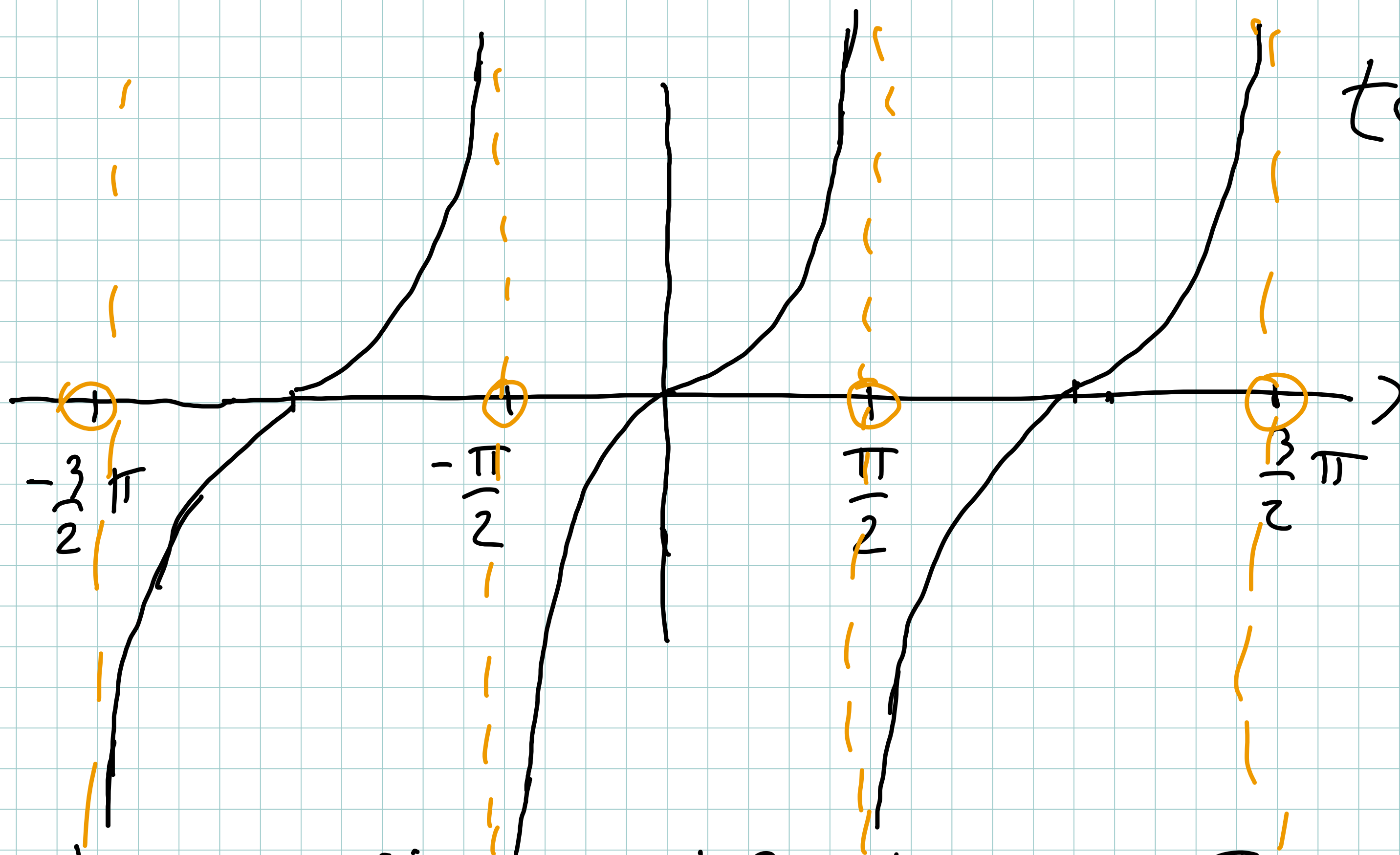
$\sin x \quad \cos x \quad \tan x \quad \cot x \quad \sec x \quad \csc x$



$$\sin\left(x + \frac{\pi}{2}\right) = \cos x$$

$\cos x = 0$ if $x = \frac{\pi}{2} + \pi k \quad k = 0, \pm 1, \pm 2, \pm 3 \ldots$

tan(x)



$\tan x = \dfrac{\sin x}{\cos x}$ defined on $x \neq \frac{\pi}{2} + k\pi$

If [redacted] ($\sin x$) $= \frac{1}{2}$ what could [redacted] ($\cos x$) be ? What could x be ?

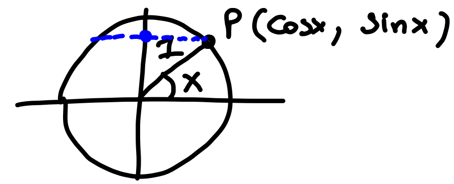
$\cos^2 x + \sin^2 x = 1$

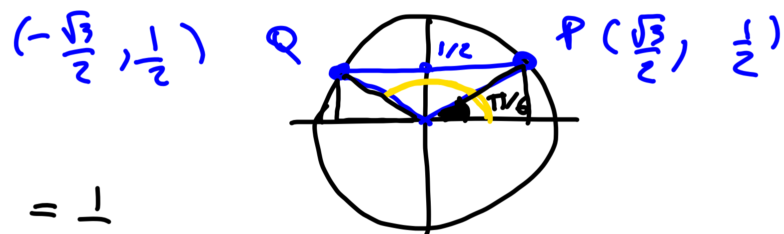
($\cos^2 x$ means $(\cos x)^2$)

$\cos^2 x + \left(\frac{1}{2}\right)^2 = 1$

$\cos^2 x = 1 - \frac{1}{4} = \frac{3}{4}$

$\cos x = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$





$\sin x = \frac{1}{2}$

From table $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

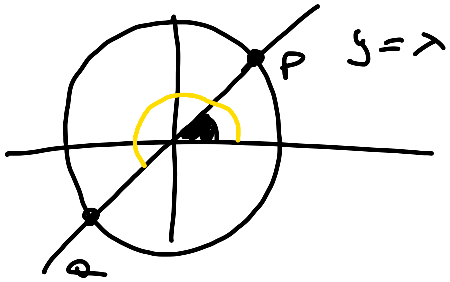
$x = \frac{\pi}{6}$

$$x = \frac{\pi}{6} + 2\pi k$$

$x = \pi - \frac{\pi}{6} = \frac{5}{6}\pi$

$$x = \frac{5}{6}\pi + 2\pi k$$

Solve $\sin \theta = \cos \theta$



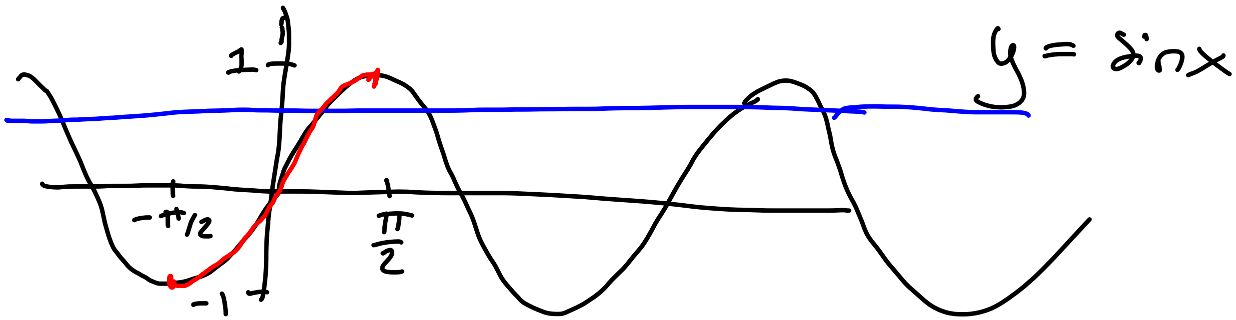
$\theta = \frac{\pi}{4}$, $\frac{5}{4}\pi$ but also

$\frac{\pi}{4} + 2\pi k$, $\frac{5}{4}\pi + 2\pi k$ so

$$\boxed{\frac{\pi}{4} + \pi k}$$

$k = 0, \pm 1, \pm 2, \ldots$

arcsin x

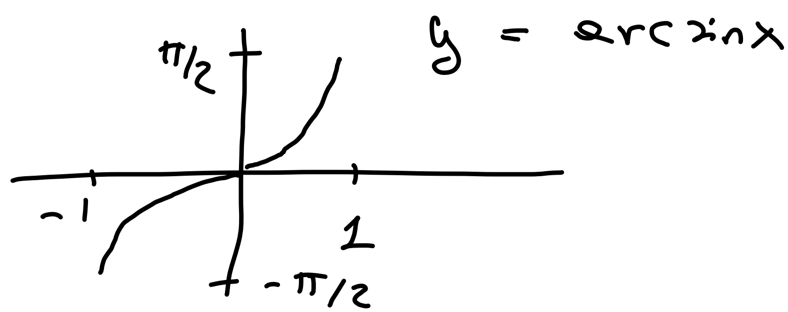


Restrict domain to $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Inverse of sin x restricted to this domain is arcsin x or $\sin^{-1} x$, it has

DOMAIN : $-1 \le x \le 1$

RANGE : $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$



Solve $\sin(x) = 0.7$ Find ALL solutions

$\arcsin(\sin x) = \arcsin(0.7)$

$x = \arcsin(0.7) = 0.77$ principal

$$\boxed{x = 0.77 + 2\pi k}$$

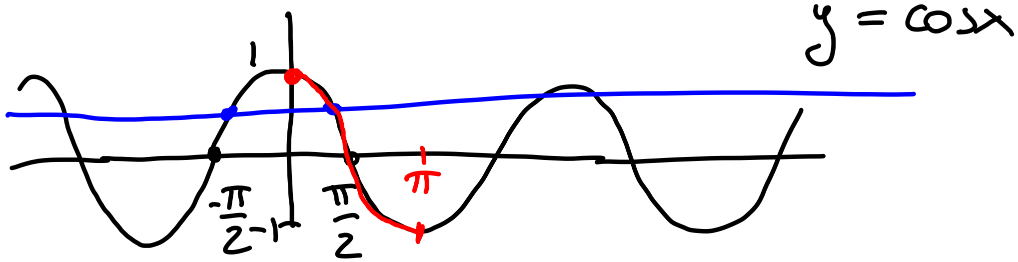
$x = \pi - 0.77 = 2.37$ symmetric

$$\boxed{x = 2.37 + 2\pi k}$$

How to solve $\sin x = c \quad (-1 \leq c \leq 1)$

1. $x_1 = \arcsin c$. This is the principal solution. It is an angle $-\frac{\pi}{2} \leq x_1 \leq \frac{\pi}{2}$
2. All values $x_1 + 2\pi k,\ k = 0, 1, 2, \cdots, -1, -2 \cdots$ are also solutions.
3. $x_2 = \pi - x_1$ is the symmetric solution. It is an angle $\frac{\pi}{2} \leq x_1 \leq \frac{3\pi}{2}$
4. All values $x_2 + 2\pi k,\ k = 0, 1, 2, \cdots, -1, -2 \cdots$ are also solutions.

arccos x



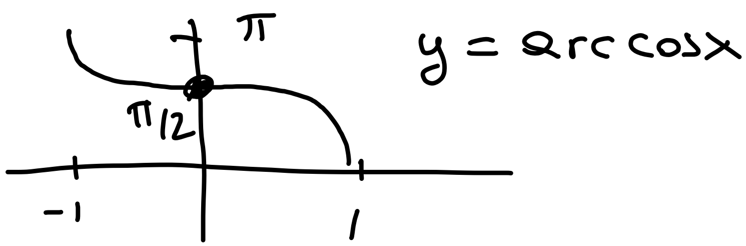
Restrict domain to $0 \le x \le \pi$

Inverse of $\cos x$ over $0 \le x \le \pi$ is $\arccos x$ or $\cos^{-1} x$

it has

Domain $-1 \le x \le 1$

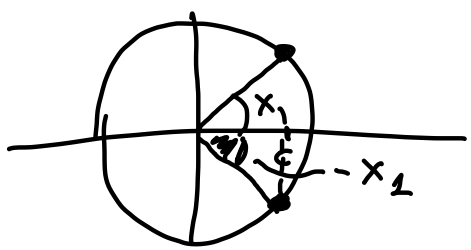
Range $0 \le y \le \pi$



$\arccos 0 = \frac{\pi}{2}$ because $\cos \frac{\pi}{2} = 0$

How to solve $\cos x = c \quad (-1 \leq c \leq 1)$

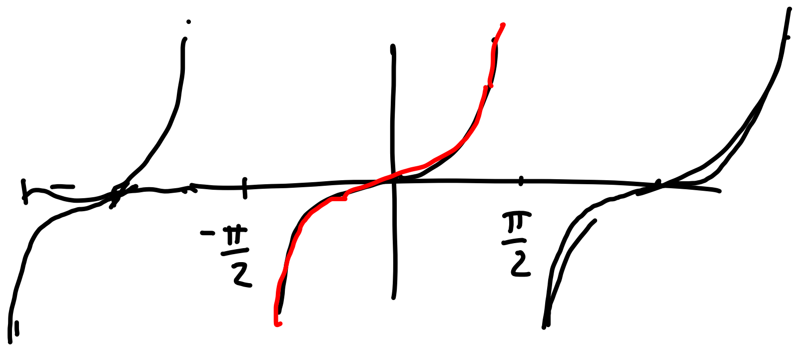
1. x_1 =arc cos c . This is the principal solution. It is an angle $0 \leq x_1 \leq \pi$
2. All values $x_1 + 2\pi k$, $k = 0, 1, 2, \cdots, -1, -2 \cdots$ are also solutions.
3. $x_2 = -x_1$ is the symmetric solution. It is an angle $-\pi \leq x_1 \leq 0$
4. All values $x_2 + 2\pi k$, $k = 0, 1, 2, \cdots, -1, -2 \cdots$ are also solutions.



$\cos x_1 = c$

$\cos(-x_1) = c$

arctan x

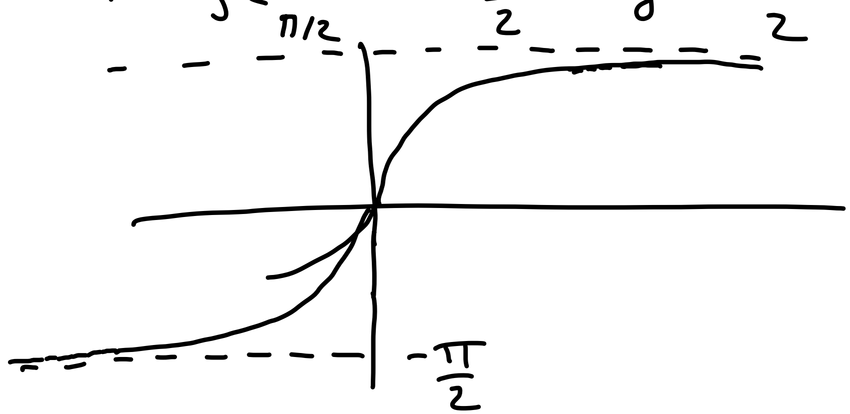


$y = \tan x$

The inverse of $\tan x$ restricted to $-\frac{\pi}{2} < x < \frac{\pi}{2}$ is called $\arctan x$ or $\tan^{-1} x$. It has

Domain $-\infty < x < +\infty$

Range $-\frac{\pi}{2} < y < \frac{\pi}{2}$



$y = \arctan x$

How to solve $\tan x = c$

1. $x_1 = \arctan\ c$. It is an angle $-\frac{\pi}{2} \leq x_1 \leq \frac{\pi}{2}$
2. All values $x_1 + \pi k,\ k = 0, 1, 2, \cdots, -1, -2 \cdots$ are also solutions.