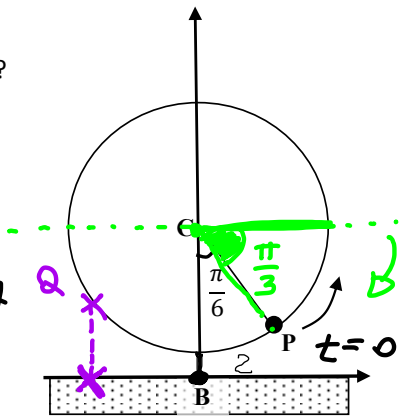
Lesson 20

Read Chapter 17

Problems involving two triangles

Spring 2012 Final $\omega = \frac{2\pi}{1.5} = \frac{4\pi}{3}$

Problem 6. (16 pts) Percy is riding on a ferris wheel of radius 50 feet, whose center C is 52 feet above ground. The wheel rotates at a constant rate in the direction shown by the arrow, taking 1.5 minutes for each full revolution. The wheel starts turning when Percy is at the point P, making an angle of $\frac{\pi}{6}$ radians with the vertical, as shown. *(Make sure your calculator is in radian mode)*



a) (5 pts) How high is Percy above ground when the wheel starts turning?

$x = 50\cos\left(\frac{4\pi}{3}t - \frac{\pi}{3}\right)$

$y = 52 + 50\sin\left(\frac{4\pi}{3}t - \frac{\pi}{3}\right)$

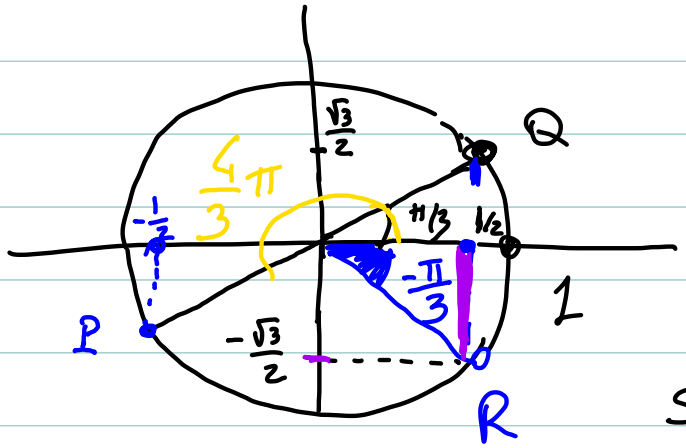
$y(0) = 52 + 50\cdot\sin\left(-\frac{\pi}{3}\right) = 52 - \frac{50\sqrt{3}}{2} = 8.7$

b) (4 pts) Impose a coordinate system **with the origin at the base point B**. What is the equation of the line CP?

c) (7 pts) Percy drops his ice cream cone 1.25 minutes after the wheel starts moving. If the cone falls straight down from Percy's position at that time, where does it land with respect to the base point B?

$x(1.25) = 50\cos\left(\frac{4\pi}{3}\,1.25 - \frac{\pi}{3}\right) =$

$50\cos\left(\frac{4}{3}\pi\right) = 50\left(-\frac{1}{2}\right) = -25$



$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

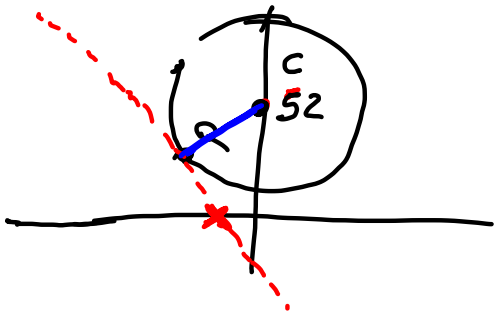
$\cos\left(\frac{4}{3}\pi\right) = -\frac{1}{2}$

At time $t = 1.25$ min Percy throws a rock. The rock travels along the line tangential to Percy's position. Where does the rock land ?

$$y(1.25) = 52 + 50\sin\left(\frac{4\pi}{3}\,1.25 - \frac{\pi}{3}\right) \simeq 8.7$$

Q $(-25, 8.7)$

C $(0, 52)$



Find slope CQ $\quad \frac{52 - 8.7}{0 - (-25)} = \frac{52 - 8.7}{25}$

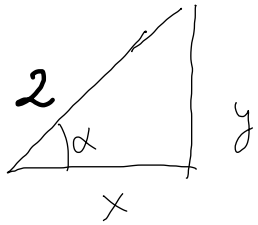
Slope of tangent $\quad m = -\frac{25}{52 - 8.7} = -0.58$

tangent $\quad y = 8.7 - 0.58(x + 25)$

Landing spot is x intercept : $\quad 0 = 8.7 - 0.58(x + 25)$

Solve for x $\quad \boxed{x = -10}$

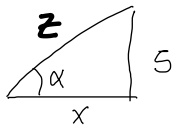
If $\alpha = 1.3$ rad, find x and y



$\sin(1.3) = \frac{y}{2}$; $y = 2 \cdot \sin(1.3) = 1.93$

$\cos(1.3) = \frac{x}{2}$; $x = 2 \cdot \cos(1.3) = 0.53$

If $\alpha = 0.5$ rad, find x and z



$$\tan(0.5) = \frac{5}{x} \quad ; \quad x = \frac{5}{\tan(0.5)} = 9.15$$

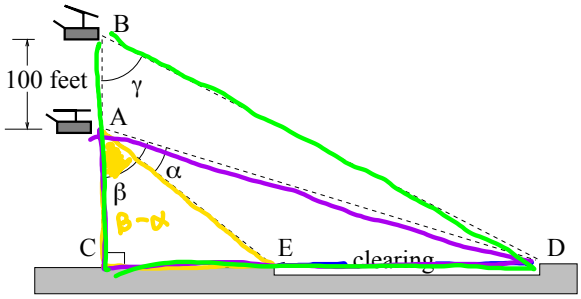
$$\sin(0.5) = \frac{5}{z} \quad ; \quad z = \frac{5}{\sin(0.5)} = 10.43$$

Win 2011 Final (hw)

7. The crew of a helicopter needs to land temporarily in a forest. They spot a flat horizontal piece of ground (a clearing in the forest) as a potential landing site, but are uncertain whether it is wide enough. They make two measurements from point A (see figure) and find $\alpha = 19^o$ and $\beta = 60^o$.

   They then rise vertically 100 feet to point B and measure $\gamma = 53^o$.

   Determine the width of the clearing.



want ED

= CD - CE

$$\frac{CE}{AC} = \tan\left((60-19)\frac{2\pi}{360}\right)$$

$$\frac{CD}{AC} = \tan\left(60\,\frac{2\pi}{360}\right)$$

$$\frac{CD}{AC+100} = \tan\left(\frac{53\ 2\pi}{360}\right)$$

1) $CE = AC \cdot \tan\left(\frac{82\pi}{360}\right) \simeq AC \cdot 0.8693$

2) $CD = AC \tan\left(\frac{120\pi}{360}\right) \simeq AC \cdot 1.7320$

3) $CD = (AC + 100) \tan\left(\frac{106\pi}{360}\right) = (AC+100)\, 1.3270$

$CE = 0.8693\ AC$

$CD = 1.7320\ AC$

$CD = 1.3270\ (AC + 100)$

2)-3) $0 = 1.7320\,AC - 1.3270\,AC + 1.3270 \times 100$

....

$AC = 327.65$

$CD = 1.7320 \cdot AC = 567.5$

$CE = 0.8693\ AC = 284.82$

$ED = CD - CE = 282.68$

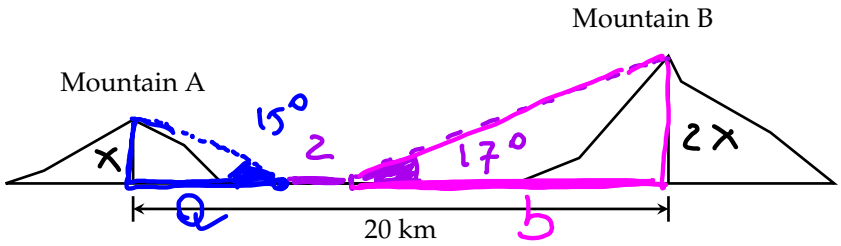
Fall 2012 Final

4. You are on a road connecting the bases of Mountain A and Mountain B.

   You look at Mountain A and measure the angle of elevation to the top of Mountain A to be $15°$.

   You then travel 2 km toward Mountain B.

   You measure Mountain B's angle of elevation from your new location to be $17°$.



   Mountain A and Mountain B are 20 km apart as shown in the figure, and Mountain B is exactly twice as tall as Mountain A.

   What is the height of Mountain A?

$$\frac{x}{a} = \tan\left(15° \frac{2\pi}{360}\right)$$

$$\frac{2x}{b} = \tan\left(17 \cdot \frac{2\pi}{360}\right)$$

$$a + 2 + b = 20$$

$$x = a \cdot 0.2679$$

$$2x = b \cdot 0.3057$$

$$a + 2 + b = 20$$

$$2a \cdot 0.2679 = b \cdot 0.3057$$

$$a = 18 - b$$

$$2(18 - b) \cdot 0.2679 = b \cdot 0.3057$$

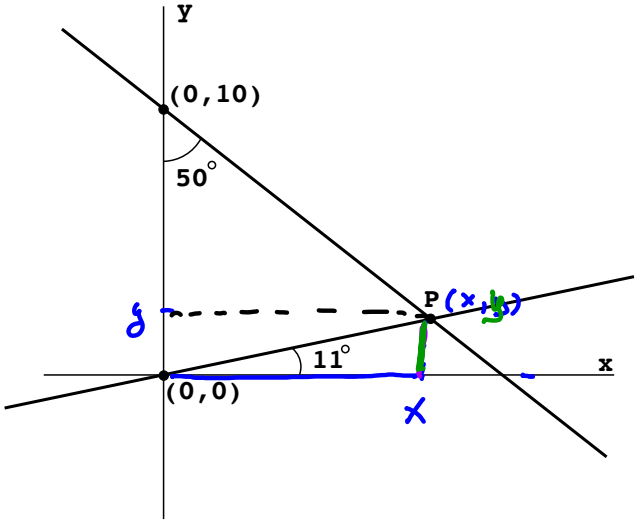
$$\therefore \quad b = 11.461$$

$$a = 6.539$$

$$x = a \cdot 0.2679 = 1.75 \text{ km}$$

Conroy Spring 2004

4. Find the coordinates of point P in the figure below.



$$\frac{y}{x} = \tan\left(\frac{11 \cdot 2\pi}{360}\right)$$

$$\frac{x}{10-y} = \tan\left(50 \cdot \frac{2\pi}{360}\right)$$

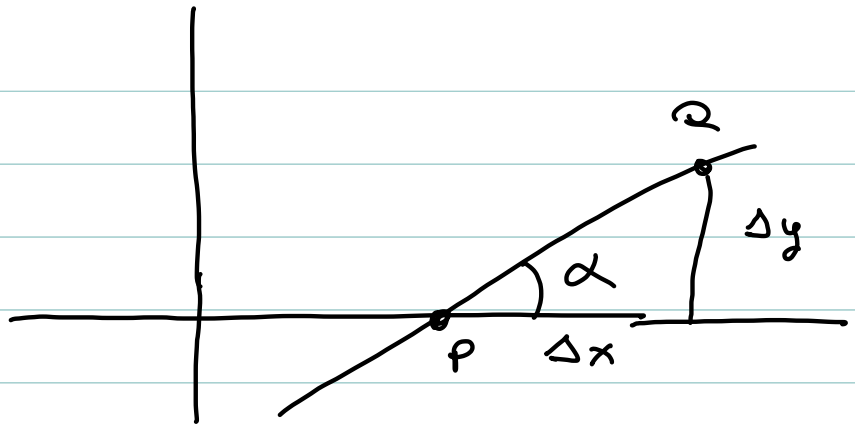
$$y = x \cdot \tan\left(\frac{11 \cdot 2\pi}{360}\right)$$

$$x = (10 - y)\tan\left(\frac{50\ 2\pi}{360}\right)$$

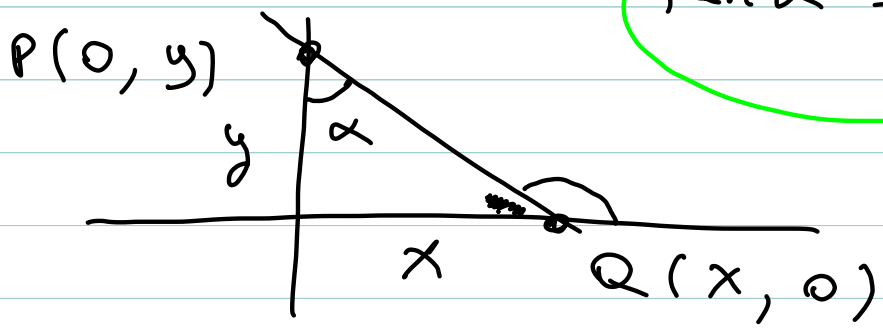
$$x = \left(10 - x\tan\left(\frac{11 \cdot 2\pi}{360}\right)\right)\tan\left(\frac{50 \cdot 2\pi}{360}\right)$$

$$x = 9.68$$

$$y = 1.88$$



slope of line is $m = \frac{\Delta y}{\Delta x} = \tan\alpha$



$\tan\alpha = \frac{x}{y}$

$\Delta y = y$

$\Delta x = -x$

$m = -\frac{y}{x} = -\frac{1}{\frac{x}{y}}$

$= -\frac{1}{\tan\alpha} = \cot\alpha$