Lesson 2

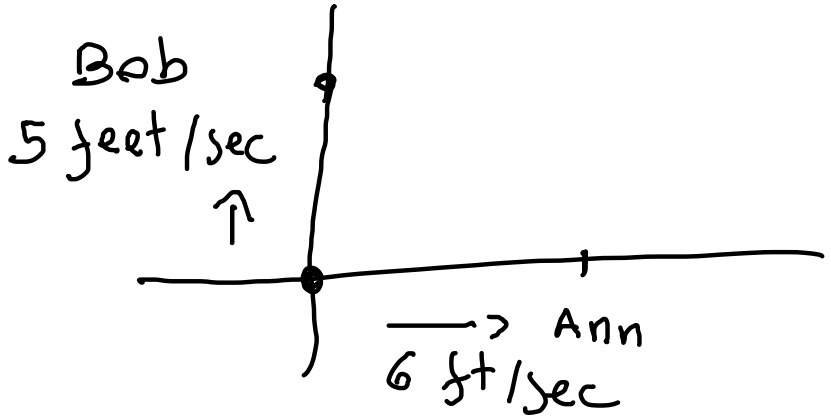
Read Chapter 3

Lines

Ann and Bob start moving at the same time from the same location. Ann moves East at 6 feet/sec. Bob moves North at 5 feet/sec.

What is the distance between Ann and Bob 10 sec later ?

When is the distance between Ann and Bob 50 feet ?



Ann $x(t) = 6t$, $y(t) = 0$ $(6t, 0)$

Bob $x(t) = 0$, $y(t) = 5t$ $(0, 5t)$

General formula $x = a + v_x(t - t_1)$ $y = b$ horizontal

$x = a$ $y = b + v_x(t - t_1)$ vertical

at $t = 10$ Ann $(60, 0)$ Bob $(0, 50)$

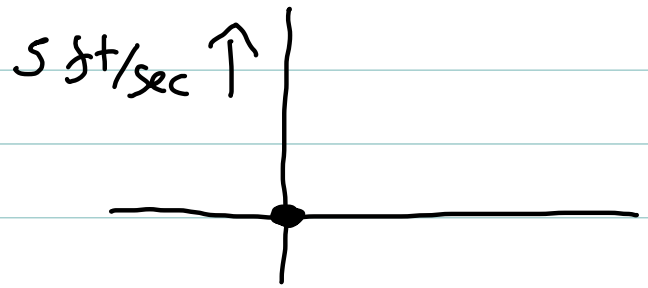
$$d = \sqrt{(60-0)^2 + (0-50)^2} = \sqrt{6100} \text{ feet}$$

$$50 = \sqrt{(6t)^2 - (-5t)^2} = \sqrt{61\,t^2} = \sqrt{61} \cdot t$$

$$50 = \sqrt{61}\ t$$

$$t = \frac{50}{\sqrt{61}} \text{ sec}$$

What if Bob starts moving 3 sec after Ann? $t=0$ is when Ann starts moving



Bob's parametric equations:

$$x = 0$$

$$y = \begin{cases} 0 & 0 \le t < 3 \\ 5(t-3) & t \ge 3 \end{cases}$$

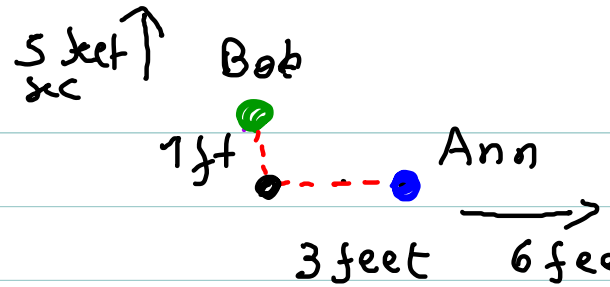
Algebra question

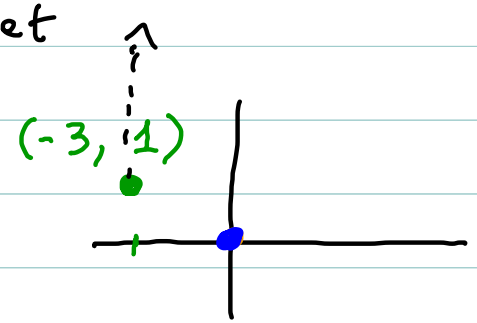
Is $\sqrt{x^2} = x$? No only if $x \geq 0$

if $x = 2$ $\sqrt{2^2} = \sqrt{4} = 2$

if $x = -2$ $\sqrt{(-2)^2} = \sqrt{4} = 2$ $(-(-2)$ or $|-2|)$

$\sqrt{x^2} = |x|$





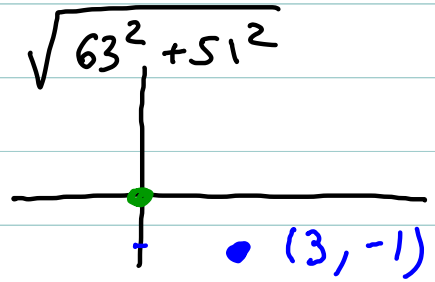
Choice 1 : origin at •

Ann : $x = 6t$, $y = 0$ $(6t, 0)$

Bob : $x = -3$, $y = 1 + 5t$ $(-3, 1+5t)$

distance after 10 sec Ann $(60, 0)$ Bob $(-3, 51)$

distance $d = \sqrt{(60-(-3))^2 + (0-51)} = \sqrt{63^2 + 51^2}$



Choice 2 : origin at •

Ann : $x = 3 + 6t$, $y = -1$

Bob : $x = 0$, $y = 5t$

Ann $(63, -1)$ Bob $(0, 50)$

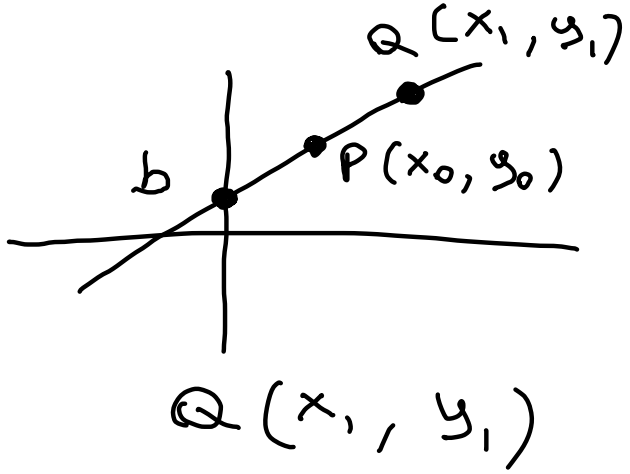
distance after 10 sec : $d = \sqrt{63^2 + (-1-50)^2} = \sqrt{63^2 + 51^2}$

Lines equations

$Ax+By+C=0$

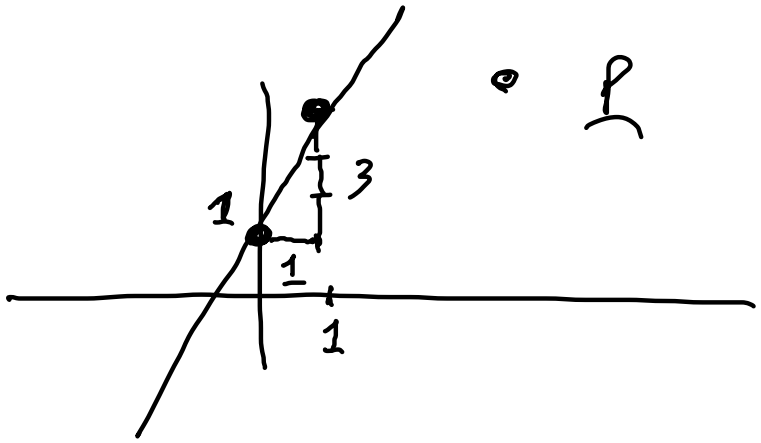
$f(x) =$
$y = mx + b$

$y = y_1 + m(x - x_1)$



$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\Delta y}{\Delta x}$$

Graph $y = 3x + 1$. Is $P(3, 4)$ on this line ?



$4 = 3 \cdot 3 + 1$? No

So P is not on line

Useful facts about lines

1. Two lines $L1 : y = m_1x + b_1$ and $L_2 : y = m_2x + b_2$ are parallel iff $m_1 = m_2$.
2. Two lines $L1 : y = m_1x + b_1$ and $L_2 : y = m_2x + b_2$ are perpendicular iff $m_1 = -\frac{1}{m_2}$ $m_2 = -\frac{1}{m_1}$
3. The slope of the line through the points (x_0, y_0) and (x_1, y_1) is $m = \frac{y_1 - y_0}{x_1 - x_0}$ if $x_0 \neq x_1$ (1, 2) (1, 5)
4. The equation of the line through point P=(x_0, y_0) and Q=(x_1, y_1) is $y = y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x - x_0)$ if $x_0 \neq x_1$ and is $y = x_1$ if $x_0 = x_1$.
5. The equation of a line through $P(x_0, y_0)$ with slope m is $y = y_0 + m(x - x0)$

Find the equation of the line through $P(1,2)$ and parallel to the line $4x - 2y + 10 = 0$

solve for y $\quad -2y = \frac{-4x-10}{-2}$

$y = 2x + 5$

m slope

$y = 2 + m(x-1)$ $\quad$ Figure out m $\quad m = 2$

$$\boxed{y = 2 + 2(x-1)}$$

Find the equation of the line through $P(1, 2)$ and perpendicular to the line $4x - 2y + 10 = 0$

$y = 2x + 5$

$y = 2 + m(x-1)$ $\qquad$ $m = -\frac{1}{2}$

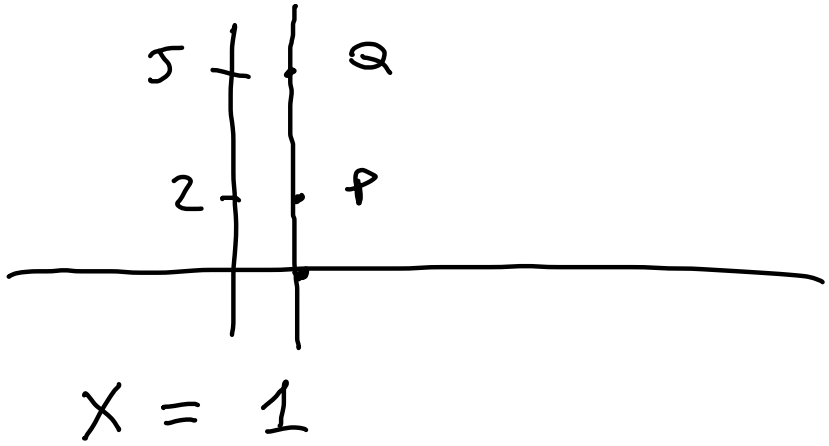
$\boxed{y = 2 - \frac{1}{2}(x-1)}$

Find the equation of the line through $P(2,2)$ and $Q(1,5)$

$$m = \frac{5-2}{1-2} = -3$$

$$y = 5 - 3(x-1) \quad \text{or}$$
$$y = 2 - 3(x-2)$$

Find the equation of the line through $P(1,2)$ and $Q(1,5)$



Decide if the lines

$$y = 2x + 1$$

$$y = x - 2$$

intersect or not. If they do find their intersection.

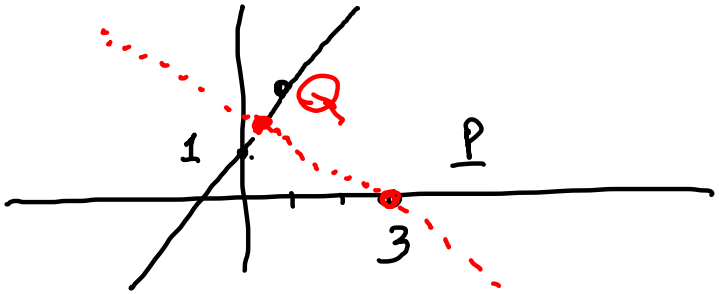
$x - 2 = 2x + 1$

$-3 = x$

$y = -3 - 2 = -5$

$P(-3, -5)$

Find the point on the line $y = 2x + 1$ that is closest to the point $P(3, 0)$



1) Find line through $P(3,0)$, $\perp$ $y=2x+1$

$y = m(x-3)$ $\quad m = -\frac{1}{2}$

$y = -\frac{1}{2}(x-3)$