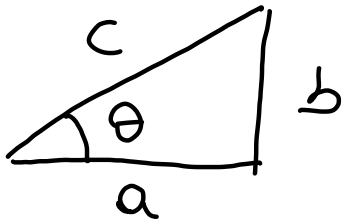
Lesson 19

Read Chapter 17

Trigonometry

Definitions for angles $0 < \theta < \frac{\pi}{2}$



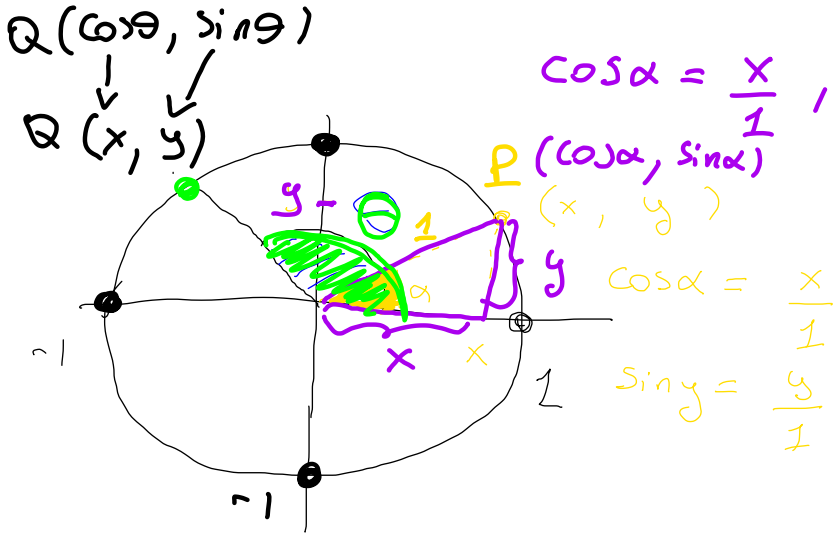
$\sin\theta = \frac{b}{c}$

$\cos\theta = \frac{a}{c}$

$\tan\theta = \frac{b}{a}$

Unit circle approach

Angle θ is measured from x axis, counter clockwise

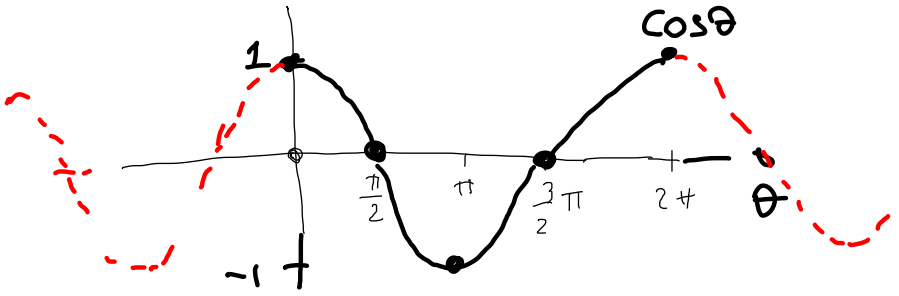


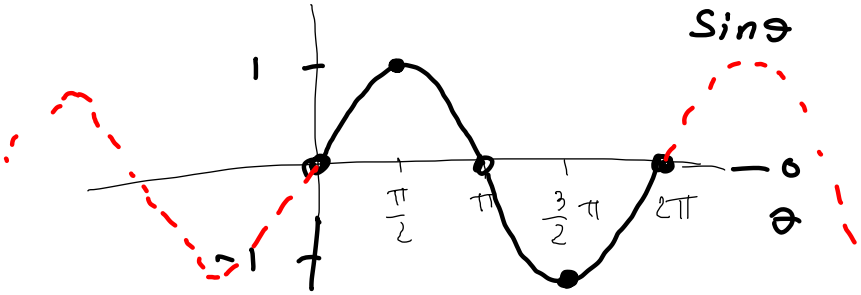
$\cos\theta$ = x coordinate of Q

$\sin\theta$ = y coordinate of Q

θ	$\cos\theta$	$\sin\theta$
0	1	0
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3}{2}\pi$	0	-1
2π	1	0

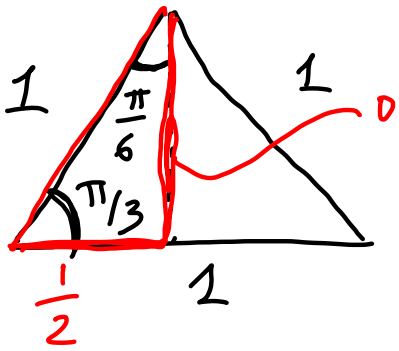
$\tan\theta = \frac{\sin\theta}{\cos\theta}$



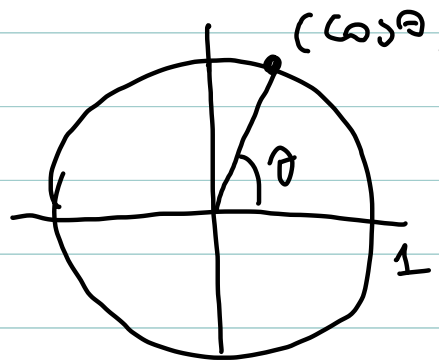


	x	$\sin x$	$\cos x$	$\tan x$
	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
	$\frac{\pi}{2}$	1	0	DNE

$= \frac{\sin x}{\cos x}$



$0 \quad \frac{\sqrt{3}}{2} \cdot 1$

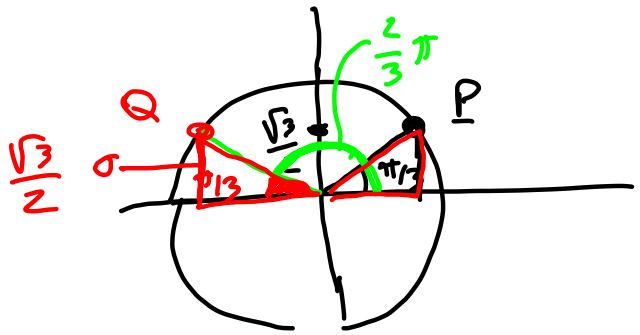


unit circle $x^2 + y^2 = 1$

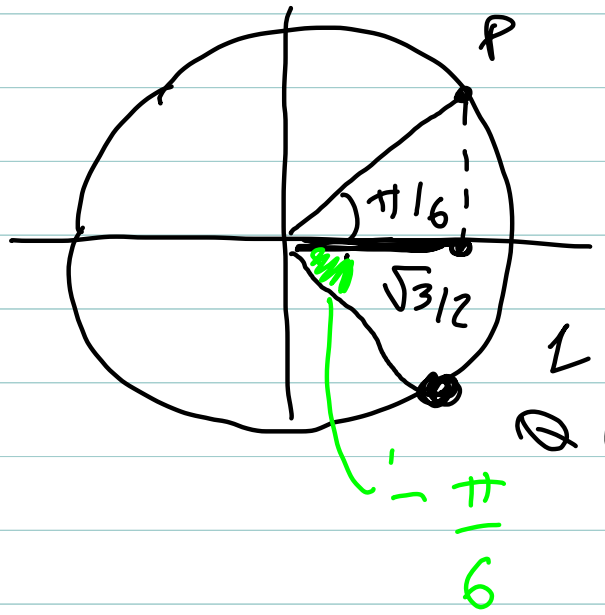
$\cos^2\theta + \sin^2\theta = 1$

Compute $\sin(\frac{2}{3}\pi)$ and $\cos(-\frac{\pi}{6})$

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$



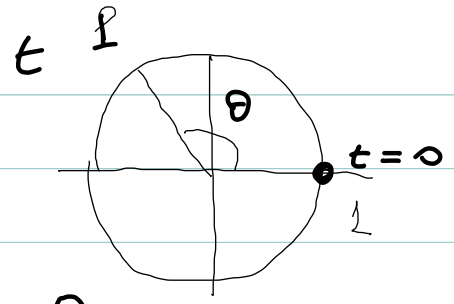
$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$



$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

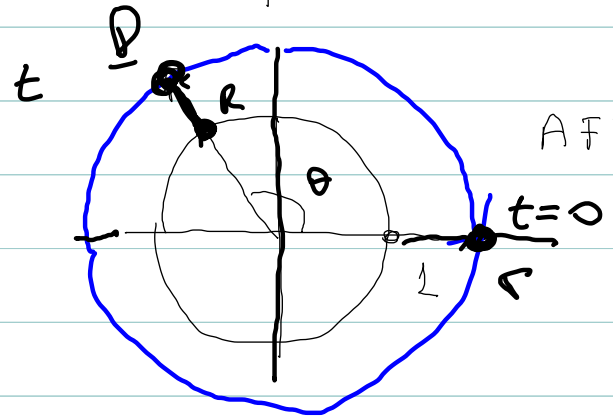
$\theta = \omega \cdot t$



AFTER TIME t at

$x = \cos\theta = \cos(\omega t)$

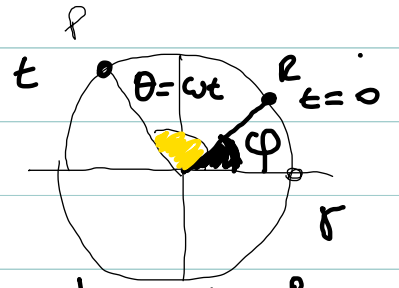
$y = \sin\theta = \sin(\omega t)$



AFTER TIME t at

$x = r\cos(\omega t)$

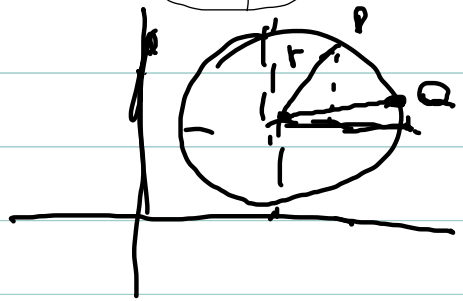
$y = r\sin(\omega t)$



AFTER TIME t at

$x = r\cos(\theta+\varphi) = r\cos(\omega t+\varphi)$

$y = r\sin(\theta+\varphi) = r\sin(\omega t+\varphi)$



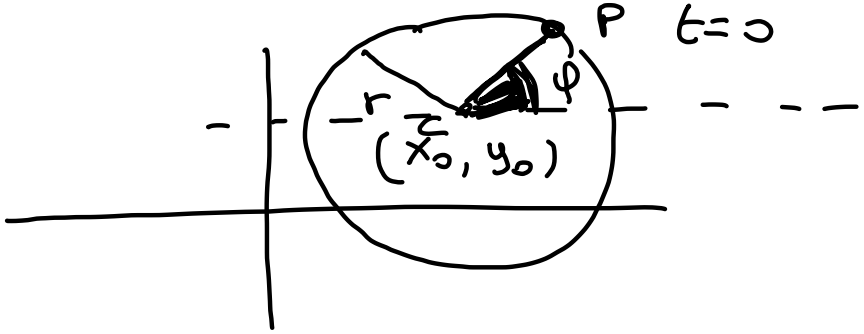
AFTER TIME t at

Parametric equations of motion for uniform circular motion

The parametric equation of motion of an object that moves on a circle of radius r centered at $C(x_0, y_0)$ with angular velocity ω and starting at P are

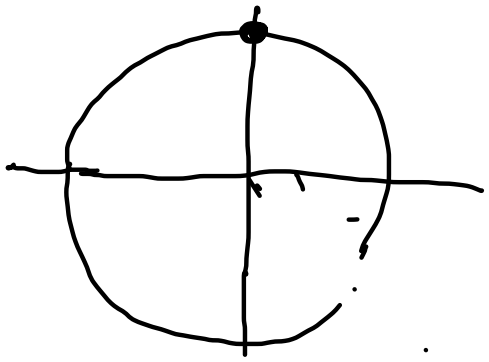
$$x = x_0 + r\cos(\omega\, t + \phi)$$

$$y = y_0 + r\sin(\omega\, t + \phi)$$

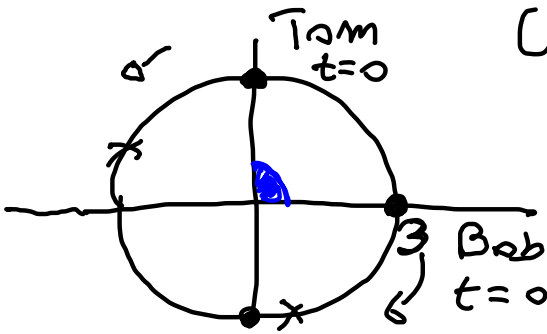


Tom is running in the counterclockwise direction around a circular lake of radius 3 mi. His linear speed is 5mph. ~~How long does it take him to run around the lake once~~ ? ~~He~~ He starts running from the Northernmost point on the lake, ~~how long does it take him to reach Q~~?

From last time $\omega_T = \frac{v}{r} = \frac{5}{3}$ rad/hour



Bob is running in the clockwise direction around the same lake. His angular speed is π rad/hr. He starts running from the Easternmost point on the lake at the same time Tom starts. What is the distance between Bob and Tom 30 min after they start running ?



$\omega_T = \frac{5}{3}$ rad/hr

$\omega_B = -\pi$ rad/hr

Tom $x = 3\cos\left(\frac{5}{3}t + \frac{\pi}{2}\right)$ for $t = \frac{1}{2}$ Tom at $(-2.22, 2.02)$

$y = 3\sin\left(\frac{5}{3}t + \frac{\pi}{2}\right)$

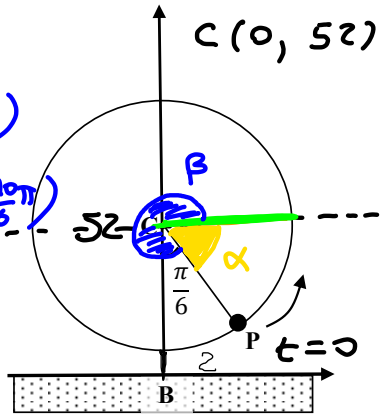
Bob $x = 3\cos(-\pi t)$ for $t = \frac{1}{2}$ $3\cos\left(-\pi \cdot \frac{1}{2}\right) = 0$ Bob at $(0, -3)$

$y = 3\sin(-\pi t)$ $3\sin\left(-\frac{\pi}{2}\right) = -3$

$d(\text{Bob}, \text{Tom}) = \sqrt{(-2.22 - 0)^2 + (2.02 - (-3))^2} = 5.49$ mi

Spring 2012 Final $T = 1.5$ $\omega = \frac{2\pi}{T} = \frac{2\pi}{1.5} = \frac{4}{3}\pi$ rad/min

Problem 6. (16 pts) Percy is riding on a ferris wheel of radius 50 feet, whose center C is 52 feet above ground. The wheel rotates at a constant rate in the direction shown by the arrow, taking 1.5 minutes for each full revolution. The wheel starts turning when Percy is at the point P, making an angle of $\frac{\pi}{6}$ radians with the vertical, as shown. *(Make sure your calculator is in radian mode)*



a) (5 pts) How high is Percy above ground when the wheel starts turning?

$x = 50\cos\left(\frac{4}{3}\pi t - \frac{\pi}{3}\right) = 50\cos\left(\frac{4}{3}t + \frac{10}{6}\pi\right)$

$y = 52 + 50\sin\left(\frac{4}{3}\pi \cdot t - \frac{\pi}{3}\right) = 52 + 50\sin\left(\frac{4}{3}t + \frac{10\pi}{6}\right)$

$\alpha = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

$\beta = \frac{3}{2}\pi + \frac{\pi}{6} = \frac{10}{6}\pi$

b) (4 pts) Impose a coordinate system **with the origin at the base point B**. What is the equation of the line CP?

We'll finish next time.

c) (7 pts) Percy drops his ice cream cone 1.25 minutes after the wheel starts moving. If the cone falls straight down from Percy's position at that time, where does it land with respect to the base point B?