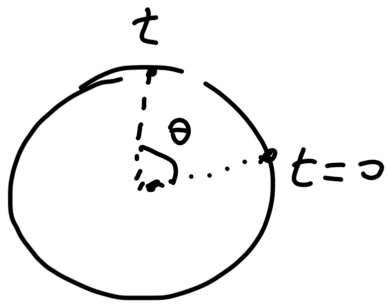
Lesson 18

Read Chapter 16

Circular motion

A rotating sprinkler r reaches 10 m far and completes a full revolution in 5 min. How much area does it irrigate in 2 min ? How long does it take the sprinkler r to irrigate 50 square meters ?



$$\frac{\theta}{2\pi} = \frac{t}{5}$$

$$\theta = \underbrace{\frac{2\pi}{5}}_{\omega} \cdot t$$

From last time

Uniform circular motion

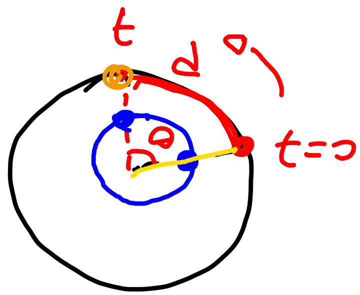
T. Period: time it takes to go around circle once

ω. Angular velocity, measures how fast angles are swept. Units rad/time, RPM

use this

v. Linear velocity, measures how fast distances (on circumference) are covered. Units distance/time

Uniform circular motion formulas



$$\overbrace{d = r\,\theta}^{\text{arc length}} = r\underbrace{\omega t}_{\theta}$$

$$\|$$

$$v\cancel{t} = r\,\omega \cdot \cancel{t}$$

$$d = vt$$

$$\theta = \omega\, t$$

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$v = \omega\, r$$

$$1RPM = 2\pi \text{ rad } / \text{min}$$

An object moves around a circle of radius 10 ft with $\omega = 12$ RPM. Find its linear velocity in feet/sec. How many radians does the object turn in 3 sec ? What distance does it cover in 3 min ?

$$\omega = 12 \cdot 2\pi \ \frac{\text{rad}}{\text{min}}$$

$$v = \omega \cdot r$$

$$v = \frac{\cancel{12}^{2} \cdot 2\pi \ \text{rad}}{\cancel{60} \ \text{sec}} \cdot 1\cancel{0} \ \cancel{\text{ft}} = \boxed{4\pi \ \text{feet}/\text{sec}}$$

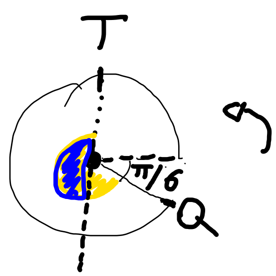
$$\theta = \omega \cdot t$$

$$\theta = \frac{12 \cdot 2\pi}{60} \cdot 3 = \boxed{\frac{6}{5}\pi \ \text{rad}}$$

$$d = v\,t$$

$$d = 4\pi \cdot 3 \cdot 60 = \boxed{720\pi \ \text{feet}}$$

Tom is running in the counterclockwise direction on a path around a circular lake of radius $r = 3$ mi. His linear speed is 5 mph. How long does it take Tom to run a full circle ? Tom starts at the Northernmost point on the path. How long does it take him to reach location Q ? (see picture)



$$T = \frac{2\pi}{\omega}$$

$$v = \omega \cdot r$$

① $\omega = \frac{v}{r} = \frac{5}{3}\,\frac{\cancel{mi}}{h} \cdot \frac{1}{\cancel{mi}} \cdot rad$

② $T = \frac{2\pi}{\frac{5}{3}} = 2\pi \cdot \frac{3}{5} = \frac{6\pi}{5}\,hr$

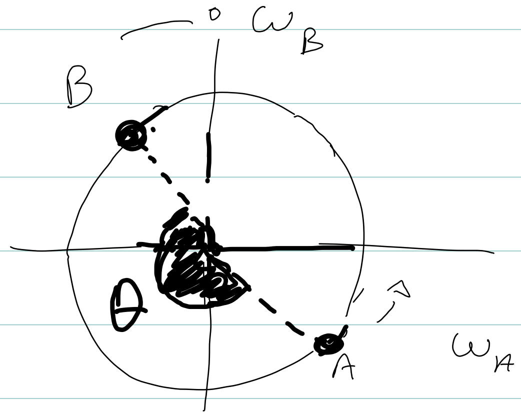
$$\alpha = \omega \cdot t$$

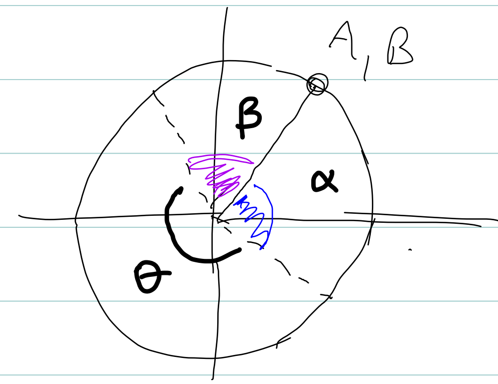
$$\pi + \frac{\pi}{2} - \frac{\pi}{6} = \frac{5}{3}\,t$$

$$\frac{6+3-1}{6}\,\pi \cdot \frac{3}{5} = t$$

$$\frac{\cancel{8}\,4}{\cancel{6}\,\cancel{2}} \cdot \frac{\cancel{3}}{5} \cdot \pi \; hr = t$$

Two people running in opposite directions





A and B pass each other for the first time when

$$\underbrace{\omega_A t}_{\alpha} + \underbrace{\omega_B t}_{\beta} + \theta = 2\pi$$

A and B pass each other for the second time when

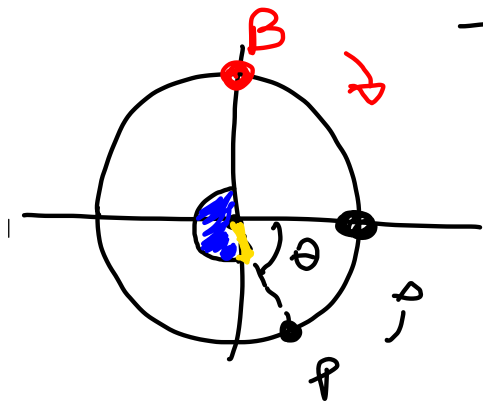
$$\omega_A t + \omega_B t + \theta = 2\pi + 2\pi$$

Conroy Mid 2
Spring 2013

2. Akira is running around a circular track. Akira runs counterclockwise. From where he starts, it takes him 22 seconds to reach the easternmost point of the track. It takes him 95 seconds to run one complete lap.

SKIP ~~(a)~~ Let r stand for the radius of the track. With a coordinate system imposed so that the origin is at the center of the track, express Akira's x- and y- coordinates as functions of the time, t, since he started running (your expressions will involve r).

Find where Akira starts (angle)



$T = 95$ $\qquad \frac{95}{4} = 23.75$

$\omega = \frac{2\pi}{95}$ rad/sec

$\theta = \omega \cdot t \qquad = \frac{2\pi}{95} \cdot 22$

(b) Bob starts running at the same time as Akira. Bob starts from the northernmost point and runs clockwise. Bob and Akira pass each other for the first time after 26 seconds. How long does Bob take to run one lap of the track?

$T_B = \frac{2\pi}{\omega_B}$

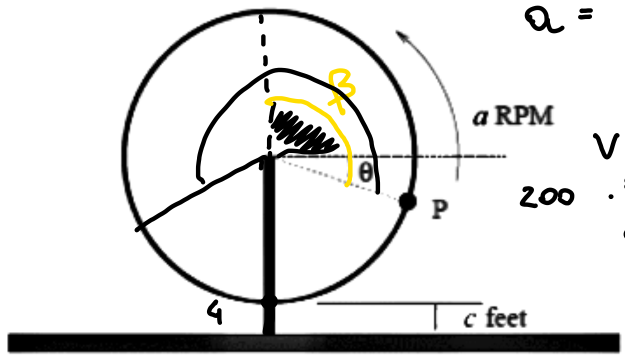
$\frac{2\pi}{95} \cdot 26 + \omega_B \cdot 26 + \overbrace{\pi + \frac{\pi}{2} - \frac{2\pi}{95} \cdot 22}^{\theta} = 2\pi$

$\frac{2\pi}{95} \cdot 4 + \omega_B \cdot 26 = \frac{\pi}{2}$; $\quad \omega_B = \dfrac{\frac{\pi}{6} - \frac{2\pi}{95} 4}{26}$

$T = \frac{2\pi}{\omega_B}$

UWAPreCalc1 16.P.005. (2172427) (Add) -- view 27m

John has been hired to design an exciting carnival ride. Tiff, the carnival owner, has decided to create the world's greatest ferris wheel. Tiff isn't into math; she simply has a vision and has told John these constraints on her dream: (i) the wheel should rotate counterclockwise with an angular speed of a = 12 RPM; (ii) the linear speed of a rider should be 200 mph; (iii) the lowest point on the ride should be c = 4 feet above the level ground.



$a = 12 \cdot 2\pi \frac{\text{rad}}{\text{min}}$; $v = 200 \frac{\text{mi}}{\text{hr}}$

$v = \omega r$

$200 \cdot \frac{5280 \text{ feet}}{60 \text{ min}} = 12 \cdot 2\pi \frac{\text{rad}}{\text{min}} \cdot r$

$\frac{200 \cdot 5280}{60 \cdot 24\pi} = r$

(a) Find the radius of the ferris wheel. (Round your answer to two decimal places.)

[ ] [redacted] ft

(b) Once the wheel is built, John suggests that Tiff should take the first ride. The wheel starts turning when Tiff is at the location P, which makes an angle θ with the horizontal, as pictured. It takes her 1.4 seconds to reach the top of the ride. Find the angle θ. (Round your answer to two decimal places.)

θ = [ ] [redacted] rad

$\pi/12 + \theta$; $\beta = \frac{12 \cdot 2\pi \text{ rad}}{60 \text{ sec}} \cdot 1.4$; $\theta = \frac{24\pi}{60} \cdot 1.4 - \frac{\pi}{2}$

(c) Poor engineering causes Tiff's seat to fly off in 8 seconds. Describe where Tiff is located (an angle description) the instant she becomes a human missile. (Give your answer as an angle measured counterclockwise from her starting point P.)

[ ] $\frac{6\pi}{5}$ rad

$\gamma = \frac{24\pi}{60} \cdot 8 = \frac{16}{5}\pi = 2\cancel{\frac{10}{5}}\pi + \frac{6}{5}\pi$

Additional Materials

Reading