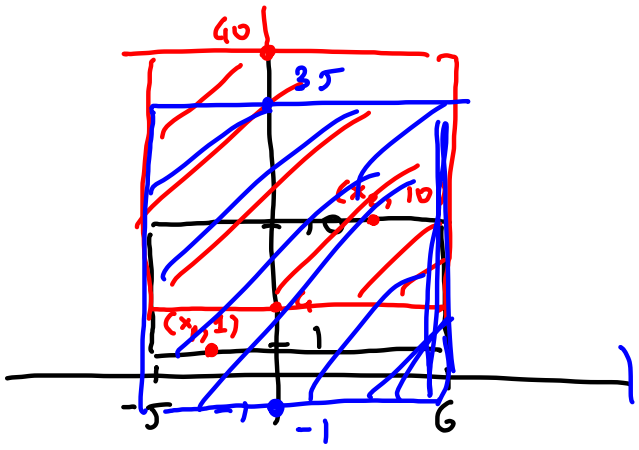
Lesson 17

Read Chapter 15

Angles. Arclenght, Area of Wedges

Suppose $g(x)$ has domain $-5 \leq x \leq 6$ and range $1 \leq y \leq 10$
What are the domain and range of $4g(x) - 5$?

$= h(x)$



① $y = 4 \cdot g(x)$ vertical scaling by a factor of 4 dilate rectangle

② $y = 4 \cdot g(x) - 5$. Move rectangle down

$-5 \leq x \leq 6$

Domain of $4g(x) - 5$

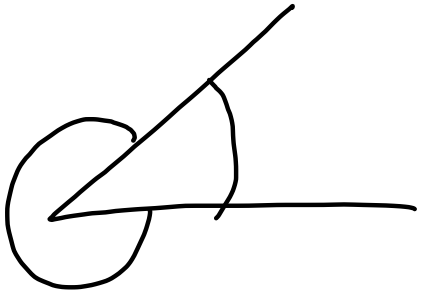
Range of $4 \cdot g(x) - 5$ $\quad -1 \leq y \leq 35$

Starting from $y = g(x)$

horizontal scaling by a factor of 4: $y = g\left(\frac{x}{4}\right)$

vertical scaling of a factor of 4: $y = 4\,g(x)$ or $\frac{y}{4} = g(x)$

An angle is the part of the plane in between two half lines starting at the same points. Angles are measured in degrees or radians. Certain precalculus/calculus formulas assume angles are measured in radians, so we often use radians as units.



$$360 \text{ deg} = 2\pi \text{ rad}$$

$$180 \text{ deg} = \pi \text{ rad}$$

$$90 \text{ deg} = \frac{\pi}{2} \text{ rad}$$

$$60 \text{ deg} = \frac{\pi}{3} \text{ rad}$$

$$45 \text{ deg} = \frac{\pi}{4} \text{ rad}$$

$$30 \text{ deg} = \frac{\pi}{6} \text{ rad}$$

$$x \text{ deg} = y\frac{360}{2\pi} \text{ rad}$$

$$y \text{ rad} = x\frac{2\pi}{360} \text{ deg}$$

If we measure angles in degrees , then $\frac{1}{60}$ of a degree is a minute and $\frac{1}{60}$ of a minute is a second. Convert 0.3 rad into deg, min, second.

$0.3 \cdot \frac{360}{2\pi} \approx 17.1887$ deg

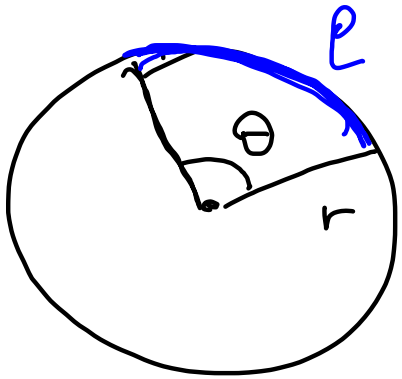
17 deg and $0.1887 \cdot 60 = 11.322$

17 deg and 11 min + $0.322 \times 60 = 19.32$

17° and $11'$ and $19''$

deg min sec

Arclength

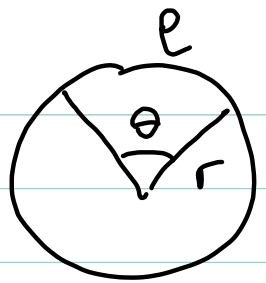


$$l = r\theta$$

θ measured in radians

$$\frac{2\pi r}{2\pi} = \frac{\ell}{\theta} \quad ; \quad r\theta = \ell$$

In degrees: $\frac{2\pi r}{360} = \frac{\ell}{\theta}$; $\frac{2\pi}{360}\theta \cdot r = \ell$

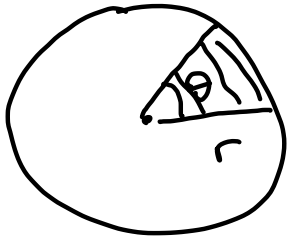


$$l = r\,\theta$$

$$\frac{l}{r} = \theta \quad \text{rad}$$

dimensionless unit

Area of wedge



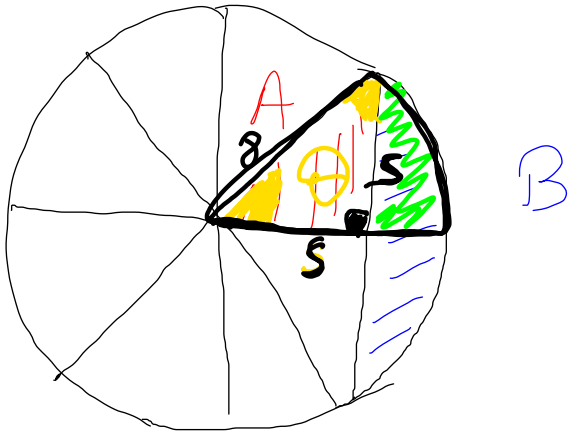
$$A = \frac{1}{2} r^2 \theta$$

θ measured in radians

$$\frac{\pi r^2}{2\pi} = \frac{A}{\theta}$$

$$\frac{1}{2} r^2 \theta = A$$

A pizza of radius 8 in is divided into 8 equal slices. Tom eats A and Bob eats B. Who eats more ?



$$\theta = \frac{2\pi}{8} = \frac{\pi}{4} \quad (45^\circ)$$

Tom

$$A = \frac{1}{2} \cdot s \cdot s = \frac{1}{2} s^2$$

pythagorean th

$$s^2 + s^2 = 8^2$$

$$2 \cdot s^2 = 64$$

$$s^2 = 32$$

$$A = \frac{1}{2} \cdot 32 = 16 \ in^2$$

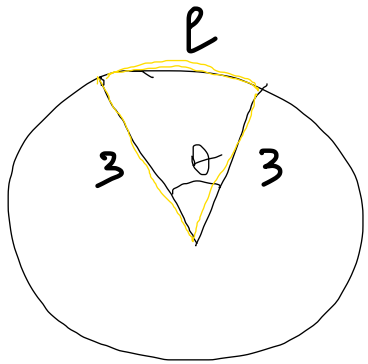
Bob

$$B = \left(\frac{1}{2} \cdot 8^2 \cdot \frac{\pi}{4} - 16 \right) \cdot 2$$

$$= 18.26 \ in^2$$

Bob eats more

Given that θ is $\frac{\pi}{6}$ and the circle has radius $r = 3$, what is the perimeter of the sector ?



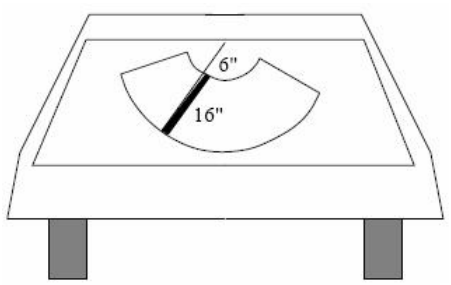
$$\ell = 3 \cdot \frac{\pi}{6} = \frac{\pi}{2}$$

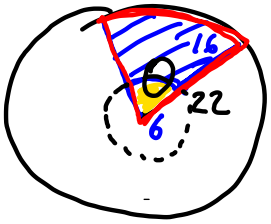
perimeter $3 + 3 + \frac{\pi}{2}$

Loveless Fall 2009

3. (12 points)

(a) The rear window wiper blade on a station wagon has a length of 16 inches. The wiper blade is mounted on a 22 inch arm, 6 inches from the pivot point (as illustrated). If the wiper turns through an angle of 105°, how much area is swept clean?





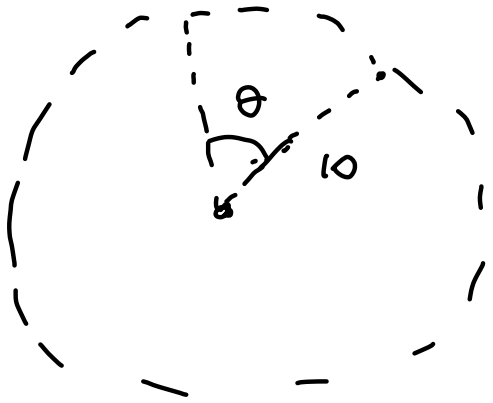
$105 \cdot \frac{2\pi}{360} = \theta$

~~(b) If $f(x) = 3x + c$ and $f(f(x)) = 9x - 16$, find the value of c.~~

$\frac{1}{2}(22)^2 \cdot \frac{105 \cdot 2\pi}{360} - \frac{1}{2} 6^2 \cdot \frac{105 \cdot 2\pi}{360} = \frac{413}{3}\pi$

(c) Find the inverse function of $f(x) = \dfrac{(\sqrt{x}-1)^2}{3}$ when restricted to the domain $0 \le x \le 1$.

A rotating sprinkler reaches 10 m far and completes a full revolution in 5 min. How much area does it irrigate in 2 min ? How long does it take the sprinkler to irrigate 60 square meters ?



(1) What angle does the sprinkler move in t min ?

$$\frac{2\pi}{5} = \frac{\theta}{t}$$

$$\frac{2\pi}{5} t = \theta$$

When $t = 2$ $\quad \theta = \frac{2\pi}{5} \cdot 2$, $\quad A = \underbrace{\frac{1}{2}}_{} \cdot \underbrace{10^2}_{r^2} \cdot \underbrace{\frac{2\pi}{5} \cdot 2}_{\theta} = 40\pi \ m^2$

$$60 = \frac{1}{2} \ 10^2 \cdot \theta$$

$$60 = \frac{1}{2} \cdot 10^2 \cdot \frac{2\pi}{5} t \qquad \text{solve for } t$$

$$\frac{6}{2\pi} = t \quad \text{min}$$